

# Program for the 2008 Front Range Applied Mathematics Student Conference

**Breakfast and Registration: 8:30 - 9:00**

**Morning Session I - Room 1312**

**9:00 - 11:00**

9:00 - 9:20	Eugene Vecharynski <i>University of Colorado at Denver</i>	The Convergence of Restarted GMRES for Normal Matrices is Sublinear
9:25 - 9:45	Adrianna Gillman <i>University of Colorado at Boulder</i>	The Numerical Performace of a Mixed-Hybrid Type Solution Methodology for Solving High-Frequency Helmholtz Problems
9:50 - 10:10	Srihari Sritharan <i>University of Wyoming</i>	Solitons to Shockwaves: Simulation and

## Plenary Address, Harry L. Swinney: 11:15 - 12:15 Room 1130

Emergence of Spatial Patterns in Physical, Chemical, and Biological Systems

Lunch: 12:15 - 1:00

### Afternoon Session I - Room 1312

1:00 - 2:15

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|-------------|--|---|
| 1:00 - 1:20 | Ben Jamroz<br><i>University of Colorado at Boulder</i>         | A Reduced PDE Model for the<br>Magnetorotational Instability            |
| 1:25 - 1:45 | Elizabeth Untiedt<br><i>University of Colorado at Denver</i>   | Fuzzy Natural Numbers   |
| 1:50 - 2:15 | Keith Wojciechowski<br><i>University of Colorado at Denver</i> | Using Pseudospectral Methods to Solve a<br>Nonlinear Transport Equation |

### Afternoon Session II - Room 1315

1:00 - 2:15

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|-------------|--|---|
| 1:00 - 1:20 | Kye Taylor<br><i>University of Colorado at Boulder</i>         | Geometric Parameterization and Denoising<br>of Manifold-Valued Data |
| 1:25 - 1:45 | Bedrich Sousedik<br><i>University of Colorado at Denver</i>    | A Recent View on the BDDC Method and its<br>Parallel Implementation |
| 1:50 - 2:15 | Christian Ketelsen<br><i>University of Colorado at Boulder</i> | Numerical Challenges in Lattice Quantum<br>Chromodynamics           |

## Plenary Speaker (11:15 - 12:15)

Emergence of Spatial Patterns in Physical, Chemical, and Biological Systems  
Harry L. Swinney, University of Texas at Austin

We consider macroscopic systems driven away from thermodynamic equilibrium by an imposed gradient, for example, a gradient in temperature, velocity, or concentration. The equation of motion for such systems is generally a nonlinear partial differential equation for the fields (e.g.,

# MORNING SESSION I

THE CONVERGENCE OF

the proposed solution methodology for solving efficiently Helmholtz problems in the mid-and-high frequency regimes.

**SOLITONS TO SHOCKWAVES:  
SIMULATION AND ANIMATION OF  
NONLINEAR WAVES ON LATTICE**

**Srihari Sritharan**

*University of Wyoming*

Using MATLAB 7.1, we calculated computational solutions to a variety of nonlinear wave equations. After discretizing the partial differential equations, we got nonlinear dynamical systems on a lattice. We utilized both an ODE and Algorithm solver method to get explicit solutions with soliton and shockwave behavior. Using a 64 particle model, we plotted and animated our results in several forms, including a linear lattice, a ring lattice (when appropriate), kinetic energy animations and phase plots. Furthermore, our use of an algorithm solver yielded threedimensional plots describing shockwave evolution. Our calculations included the use of the Linear Advection Equation, the second order wave equation, the Toda Exponential Lattice, the Inviscid Burgers Equation, and the SineGordon Equation. We finished our project with the experimentation of controls on our previously calculated lattices.

**NESTED ITERATION FIRST-ORDER  
LEAST SQUARES ON  
INCOMPRESSIBLE RESISTIVE  
MAGNETOHYDRODYNAMICS**

**James Adler**

*University of Colorado at Boulder*

Magnetohydrodynamics (MHD) is a single-fluid theory that describes Plasma Physics. MHD treats the plasma as one fluid of charged particles. Hence, the equations that describe the plasma form a nonlinear system that couples Navier-Stokes with Maxwell's equations. To solve this system, a nested-iteration-Newton-FOSLS-AMG approach is taken. The system is linearized on a coarse grid using a Newton step and is then discretized in a FOSLS functional upon which several AMG V-cycles are performed. If necessary,

another Newton step is taken and more V-cycles are done. When the linear functional has converged "enough," the approximation is interpolated to a finer grid where it is again linearized, FOSLized, and solved for. The goal is to determine the most efficient algorithm in this context. One would like to do as much work as possible on the coarse grid including most of the linearization. Ideally, it would be good to show that at most one Newton step and a few V-cycles are all that is needed on the finest grid. This talk

to compute, and it is therefore not computationally efficient to use these matrices directly to form a coarse-grid correction. Our proposed approach approximates these coarse-grid corrections by using smoothed aggregation to accurately approximate the right and left singular vectors of  $A$  that correspond to the lowest singular value. These are used to construct the interpolation and restriction operators, respectively. We present some preliminary two-level convergence theory and numerical results.

## MORNING SESSION II

### EXEMPLAR-BASED MULTIRESOLUTION, SEAM CARVING, AND THEIR APPLICATIONS TO IMAGE PROCESSING

Joseph Adams, Jonathan Olson, and  
Ryan Schilt

*University of Colorado at Boulder*

Exemplar-Based Multiresolution is a reversible decomposition of hierarchical data into a difference tree. It is constructed at each level by finding an exemplar and storing the differences between it and all other data points at that level. Since many differences are usually small, they can be removed to compress the data. By defining a multiresolution structure for an image, we can then decompose it into a difference tree. Using the difference tree, we can quickly recover an approximation to the original image using simple arithmetic. When compared with standard wavelet image compression techniques, our algorithm yields slightly worse error. However our algorithm can be used on arbitrary hierarchical structures with user-defined difference functions to which wavelets would not apply. Seam Carving is a method used to resize images by carving out only parts of the image, instead of scaling an image. An energy is defined for each pixel, and seams are removed by finding paths through the image with the least total energy. We have implemented a fast seam searching algorithm similar to Dijkstras algorithm in which computational complexity is proportional to the number of pixels in the image.

## THE EFFECT OF UNCERTAINTY IN INTERFACE LOCATION

Michael Presho

*University of Wyoming*

The study of reservoir fluid flow is a broad research topic with plenty of room for advancement. In our research we considered the effects of uncertainty with respect to the location of a composite reservoirs interface. In order to isolate the source of uncertainty we chose two idealized models in which analytical solutions could be found. We assume that the interface location follows certain statistical distribution from which a number of its realizations can be generated. A Monte Carlo simulation is then performed to obtain various statistical moments of the flow solution related to the nature of location uncertainty. In the future we hope to perform sensitivity analysis on the problem with adjoint (or other) methods.

## VARIATIONAL INTEGRATORS IN NUMERICAL MODELING OF MECHANICAL SYSTEMS

Kirk Nichols

*University of Colorado at Boulder*

Variational Integrators are a relatively unexplored numerical integration tool derived from variational calculus that are efficient in modeling dynamics. The presentation would begin with a trivial example to prove the variational integrator is worth listening to and then progressively add on more theory to increase the robustness of the variational integrator.

Our trivial example is considering a one-link pendulum. Given coordinate system  $\theta$  and fixed length  $L$ , we will derive the corresponding Lagrangian equation and the Euler-Lagrange Equation. Once at this ODE,  $\text{rogr}(e)-1(e)-r.469(e)-\text{oredthe vnm}$



center of spiral galaxies. The accretion rates of these disks, deduced from observation, requires an efficient mechanism for angular momentum extraction. The magnetorotational instability, in magnetized accretion disks, is widely believed to be the mechanism providing the necessary angular momentum transport. Taking advantage of disparate spatio-temporal scales relevant to astrophysics and laboratory experiments, one can derive a reduced PDE model for the magnetorotational instability. These reduced equations, which are characterized by a back-reaction onto the imposed local shear, can be used to analyze the nonlinear saturation of this instability and measure local angular momentum transport.

### **FUZZY NATURAL NUMBERS**

**Elizabeth Untiedt**

*University of Colorado at Denver*

This talk will explore the recent idea of fuzzy natural numbers, which represent the cardinality of a fuzzy set. The concept of fuzzy natural numbers will be extended to define fuzzy relative integers and fuzzy rational numbers. The speaker will define fuzzy prime numbers, and introduce and prove some original related theorems.

### **USING PSEUDOSPECTRAL METHODS TO SOLVE A NONLINEAR TRANSPORT EQUATION**

**Keith Wojciechowski**

*University of Colorado at Denver*

Numerical methods for time-dependent linear partial differential equations (PDEs) typically discretize the spatial derivative and use any number of time-marching schemes readily available whereas numerical methods for time-dependent, nonlinear PDEs can be highly specialized. For example, in the case of linear problems, there are implicit and explicit Euler methods, alternating-direction implicit (ADI) methods, and the Crank-Nicolson method for parabolic problems as well as the Leapfrog method, Lax-Wendroff method, and backward-difference in time method for hyperbolic problems. In the case of nonlinear problems, Runge-Kutta methods are a typical first attempt.

The accuracy with respect to the spatial derivative can be improved by either choosing high-order polynomials or finite-difference formulas or using spectral differentiation matrices. Spectral methods are implemented by approximating the spatial derivative using a global interpolant through discrete data points, then differentiating the interpolant at each point. Under favorable circumstances spectral methods boast a higher accuracy per computational cost than finite differences or finite elements (note that spectral differentiation matrices are sparsely implemented via the FFT). A nonlinear transport model for a swelling porous material is proposed and numerically solved. A pseudospectral method is implemented for the spatial derivatives while the time-stepping is executed by separating the equation into linear and nonlinear parts. The linear part is solved exactly while the nonlinear part is solved using numerical quadrature. This method is then compared to a fourth-order Runge-Kutta scheme.

## **AFTERNOON SESSION**

### **GEOMETRIC PARAMETERIZATION AND DENOISING OF MANIFOLD-VALUED DATA**

**Kye Taylor**

*University of Colorado at Boulder*

Several methods for learning a datasets underlying topological structure have been proposed that typically use local methods.

the dataset is corrupted by noise. To remove noise, one approach uses the spectral properties of the operator defined on the dataset to build a filter to denoise a signal. I will discuss several experiments involving this algorithm including removing noise from images, time series, and other low-dimensional manifolds. In addition, I will consider the consequences of applying the filter, effects of nonuniform sampling, as well as computational costs and practical issues.

### **A RECENT VIEW ON THE BDDC METHOD AND ITS PARALLEL IMPLEMENTATION**

**Bedrich Sousedik**

*University of Colorado at Denver*

The presentation covers an ongoing effort to efficiently implement the Balancing Domain Decomposition with Constraints method (BDDC) for solving large systems of equations arising from linear elasticity analyses. The BDDC method is seen as a preconditioner in PCG method. Within this framework, solution to an inexact problem is found by a direct solver. In our latest formulation of the method, decomposition of the domain just gives us a way to construct the inexact problem. It is done by relaxing most (but not all) the continuity requirements on the solution among subdomains and thus "inflating" the space where the problem is defined. We end up with a larger matrix than the one of the original problem, which is then solved exactly. However, the simple structure of this matrix makes the solution by a direct method easy and thus possible to use it for preconditioning. In finite element terminology, the larger problem is constructed by so called "partial assembly", a process that does not assemble the matrix at most of the interface nodes among subdomains. The current version of the implementation is based on the Multifrontal Massively Parallel Solver (MUMPS), an interesting open source package for solving linear equations that will be in short introduced.

### **NUMERICAL CHALLENGES IN LATTICE QUANTUM CHROMODYNAMICS**

**Christian Ketelsen**

*University of Colorado at Boulder*

Quantum Chromodynamics (QCD) is the predominant theory describing the strong interactions in the standard model of particle physics. The strong force confines quarks together inside of composite particles like protons and neutrons. Unlike particles prevalent in quantum electrodynamics (QED), the forces between quarks get stronger as the distance between the particles increases. This makes the usual asymptotic techniques employed in QED inadequate as a means of characterizing the strong force. As a result, large scale numerical simulations are necessary to model these interactions for physically realistic parameters.

The large computational obstacle in such simulations is the numerical solution of a large system of partial differential equations which we discretize on a four dimensional space-time lattice. For physically interesting parameters the resulting linear system is large, highly disordered, and near singular, making traditional iterative solution methods insufficient. A brief introduction will be given to popular discretizations of the governing equations and a new discretization based on a least squares finite element method will be presented. Methods based on smoothed aggregation multigrid will be explored for the solution of the resulting linear systems.