

Fifth Annual Front Range  
Applied Mathematics Student Conference  
March 14, 2009

**Breakfast and Registration: 8:30 - 8:55**

**Morning Session I - Room NC1605**

**9:00 - 11:00**

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|---------------|--|--|
| 9:00 - 9:20   | Je rey Larson<br><i>University of Colorado, Denver</i>     | An Implementation of Scatter Search<br>to Classify Medical Images    |
| 9:25 - 9:45   | Donald McCuan<br><i>University of Colorado, Denver</i>     | Eigensolvers for Analysis of Microarray<br>Gene Expression Data      |
| 9:50 - 10:10  | Daniel Kaslovsky<br><i>University of Colorado, Boulder</i> | Performance of Empirical Mode Decomposition<br>on Noisy Data         |
| 10:15 - 10:35 | Tim Lewkow<br><i>University of Colorado, Colo. Springs</i> | Line-Soliton Solutions of the KP Equation                            |
| 10:40 - 11:00 | Nathan Halko<br><i>University of Colorado, Boulder</i>     | A Randomized Algorithm for the Approximation<br>of Low Rank Matrices |

**Morning Session II - Room NC1603**

**9:00 - 11:00**

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|---------------|--|--|
| 9:00 - 9:20   | Matthew Nabity<br><i>University of Colorado, Denver</i>                | On the use of pivoted Cholesky<br>in the CholeskyQR algorithm  |
| 9:25 - 9:45   | Christian Ketelsen<br><i>University of Colorado, Boulder</i>           | A Transformation of the 2D Dirac Equation<br>of Quantum Electrodynamics  |
| 9:50 - 10:10  | Keith Wojciechowski<br><i>University of Colorado, Denver</i>           | A High-Order Numerical Solution to a Partial<br>Integrodi erential Equation Modeling<br>a Swelling Porous Material |
| 10:15 - 10:35 | Anil Damle, Geo rey Peterson<br><i>University of Colorado, Boulder</i> | Finding the Eigenstructure of Isoceles Triangles<br>Using McCartin's Method  |
| 10:40 - 11:00 | Elizabeth Untiedt<br><i>University of Colorado, Denver</i>             | A Survey of Applications for the<br>Mathematics of Fuzzy Sets  |

**Break: 11:00 - 11:15**

## Plenary Address: 11:15 - 12:15, Plaza M205

Mark Newman, *University of Michigan*

Epidemics, Erdos numbers, and the Internet: The structure and function of complex networks

## Lunch: 12:15 - 1:00

### Afternoon Session I - Room NC1605

1:00 - 3:00

1:00 - 1:20	Michael Presho <i>University of Wyoming</i>	The Multiscale Finite Element Method
1:25 - 1:45	Kye Taylor <i>University of Colorado, Boulder</i>	Automatic Detection and Identification of Seismic Waves
1:50 - 2:10	Christian Hampson <i>Colorado State University</i>	Characteristics of Certain Families of Random Graphs
2:15 - 2:35	Lei Tang <i>University of Colorado, Boulder</i>	Parallelization of Efficiency-Based Adaptive Local Refinement for FOSLS-AMG
2:40 - 3:00	Douglas Baldwin <i>University of Colorado, Boulder</i>	Soliton Generation and Multiple Phases in Dispersive Shock and Rarefaction Wave Interaction

### Afternoon Session II - Room NC1603

1:00 - 3:25

1:00 - 1:20	Yang Zou <i>Colorado State University</i>	Evolution of Quantitative Traits with Immigration
1:25 - 1:45	Minho Park <i>University of Colorado, Boulder</i>	A New Least Squares Based AMG
1:50 - 2:10	Doug Lipinski <i>University of Colorado, Boulder</i>	Lagrangian Coherent Structures: An Application to Jelly fish Feeding
2:15 - 2:35	Jennifer Maple <i>Colorado State University</i>	Characterization of Spatio-Temporal Complexity in Ginzburg-Landau Equations for Anisotropic System
2:40 - 3:00	Christopher Harder <i>University of Colorado, Denver</i>	A PEGM Method for the Darcy Problem
3:05 - 3:25	Ryan Kennedy <i>University of Colorado, Boulder</i>	A Framework for Pattern Recognition in Molecular Biology Data

## Afternoon Session III - Room NC1806

1:00 - 3:25

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|-------------|---|--|
| 1:00 - 1:20 | Yongli Chen, Tim Lewkow<br><i>University of Colorado, Colo. Springs</i>                       | <i>Modeling Contest, Problem B</i><br>Estimating the Impact of Wireless Communications<br>Growth on Energy Consumption and the Environment |
| 1:25 - 1:45 | Lee Rosenberg, Michelle Rendon,<br>Manuchehr Aminian<br><i>University of Colorado, Denver</i> | <i>Modeling Contest, Problem B</i><br>Modeling Energy Consumption<br>of the Current Telecommunications Structure                           |
| 1:50 - 2:10 | Anil Damle, Anna Lieb,<br>Georgy Peterson   | <i>Modeling Contest, Problem A</i><br>Pseudo-Finite Jackson Networks and T3(Net)2pAionsUni   |



randomized algorithm. Then using the basis to reduce the size of the matrix, the SVD can be computed for much less cost than working directly with the original matrix. We consider a class of matrices whose singular values decay exponentially. In this case, the rank of the matrix is much less than the dimension of the span which gives us a good setting to use randomized sampling. However, we also have a mechanism to deal with slower decay. A discussion of the algorithm will detail our adaptive rank determination and error checking procedures. Numerical examples will be presented that illustrate the robustness and precision of the resulting approximations.

been proposed. All of the variants we have found in the literature need the eigenvalue decomposition of the symmetric matrix  $A^T A$ . If we assume  $m \gg n$ , then the eigenvalue decomposition of the symmetric matrix  $A^T A$  is an  $O(n^3)$  steps which is negligible with respect to the two other operations ( $O(mn^2)$ ). However, in actuality, this step can significantly increase the overall time to solution compared to a Cholesky factorization. We propose to perform a pivoted Cholesky factorization of the matrix  $A^T A$  to cure the problem of the breakdown in Cholesky.

With this modification, CholeskyQR runs without breakdown; however the resulting matrix  $Q$

## MORNING SESSION II

### ON THE USE OF PIVOTED CHOLESKY IN THE CHOLESKY-QR ALGORITHM

Matthew Nability

*University of Colorado, Denver*

Given a nonsingular  $m$ -by- $n$  matrix  $A$  ( $m \gg n$ ), the CholeskyQR algorithm computes the QR factorization of the matrix  $A$ . In practice, it is by all means the fastest known algorithm for the QR factorization of a tall and skinny matrix and it indeed achieves (in the big  $O$  sense) lower bounds for latency, bandwidth and number of operations. It is also an extremely simple algorithm to formulate. Using Matlab notations, the algorithm reads:

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R = A'*A; R = chol(R); Q = A / R;
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Unfortunately, this algorithm is unstable and, for ill-conditioned matrix  $A$ , the resulting vectors  $Q$  do not form an orthonormal basis. ( $\|Q^T Q - I\| \approx \|A\|^{-1}$ .) And indeed for  $\kappa(A) \gg 1$ , it is likely that the algorithm breaks down in the Cholesky factorization phase. (In exact arithmetic,  $A$  nonsingular guarantees  $A^T A$  symmetric positive definite. In floating point arithmetic,  $A^T A$  is symmetric (by construction) but can be indefinite. Therefore Cholesky breaks down.)

In order for the algorithm to succeed in the Cholesky factorization phase, several variants have

least-squares finite elements and adaptive multi-level methods, respectively.

Special attention will be paid to fuzzy optimization theory, as well as to the author's own work in radiation therapy planning, heating oil distribution, and structured financial products.

## **AFTERNOON SESSION I**

### **THE MULTISCALE FINITE ELEMENT METHOD**

**Michael Presho**  
*University of Wyoming*

The study of reservoir fluid flow is a broad research topic with plenty of room for advancement. In many cases existing data is sparse, and as a result, a number of reservoir uncertainties and scales can arise. Mathematically speaking, the model equations of interest often include multi-scale variables. For the presentation I will consider two-dimensional pressure and concentration equations which describe flow in a porous medium. The heterogeneous porous medium is described through small-scale assumptions that are made on the initial permeability data, and the equations are solved using the multiscale finite element method (MsFEM). The goal of MsFEM is to incorporate the small-scale data into basis functions which can be used to treat the large scale problem. Through using MsFEM, we effectively solve on a coarse mesh while still maintaining the effects of the initial fine scale data.

### **AUTOMATIC DETECTION AND IDENTIFICATION OF SEISMIC WAVES**

**Kye Taylor**  
*University of Colorado, Boulder*

We address the problem of automatically detecting and identifying regional seismic phases (Pg, Pn, Lg, Sn) from a single-component, or a three-component, seismogram. Operating under the assumption that seismic signals organize themselves around a smooth manifold, we use discrete approximations to the manifold's vibratory modes to obtain a sparse representation for each of the seismic signals. These new representations

are not only faithful to the manifold structure influencing the data, but provide a way to analyze the data that is not cursed by the large number of samples measured in the time-domain. Once the new representations are determined, we train a classifier to first detect whether or not a seismic phase is present, and if so, assign a specific phase to the unlabeled data. Our algorithm is compared against industry standards.

### **CHARACTERISTICS OF CERTAIN FAMILIES OF RANDOM GRAPHS**

**Christian Hampson**  
*Colorado State University*

Applications of random graphs include prediction of disease and habitat invasion processes. Families of edge-product random graphs can be characterized by their probability matrix. The distributions of some attributes of these graphs

**SOLITON GENERATION AND  
MULTIPLE PHASES IN DISPERSIVE  
SHOCK AND RAREFACTION WAVE  
INTERACTION**

**Douglas Baldwin**

*University of Colorado, Boulder*

Interactions of dispersive shock (DSWs) and rarefaction waves (RWs) associated with the Korteweg-de Vries equation are shown to exhibit multiphase dynamics and isolated solitons. There are six canonical cases: one is the interaction of two DSWs which exhibit a transient two-phase solution, but evolve to a single phase DSW for large time; two tend to a DSW with either a small amplitude wave train or a finite number of solitons, which can be determined analytically; two tend to a RW with either a small wave train or a finite number of solitons; finally, one tends to a pure RW.

**AFTERNOON SESSION  
II**

**EVOLUTION OF QUANTITATIVE  
TRAITS WITH IMMIGRATION**

**Yang Zou**

*Colorado State University*

Ecological and genetical changes occur simultaneously but on different time scales and evolution is usually much slower than ecological changes. The interactions between them lead to the fast-slow dynamical system. In this paper, a model describing evolution of the mean value of a quantitative trait with a direct migration scheme is proposed, and numerical methods are applied to test assumptions and the robustness of the approximate equation. Finally, a fast-slow dynamical system, which relates the immigration to the ecological subsystem is set up and analyzed numerically.

**A NEW LEAST SQUARES BASED  
AMG**

**Minho Park**

*University of Colorado, Boulder*

In this talk we compare Brandt's BAMG with indirect Bootstrap Algebraic Multigrid (*i*BAMG) that determines the interpolation weights by collapsing F-F connections in the operator equation. Solving the linear system using least squares based AMG requires set of vectors that are results of several fine level relaxation sweeps on homogeneous equation  $Ae = 0$ . Unlike Brandt's BAMG, new method approximate all F-F connections in least squares sense. The presented numerical experiments demonstrate that the method can achieve good convergence with less vectors and relaxation sweeps.

**LAGRANGIAN COHERENT STRUCTURES**



steady oblique Hopf normal travelling interaction in an anisotropic extended system is presented. The model was derived. The system is simulated numerically using a pseudo-spectral method to compute the solution to these equations and provide a visualization tool to study the bifurcations in the system and its spatio-temporal complex behavior.

### **A PEGM METHOD FOR THE DARCY PROBLEM**

**Christopher Harder**  
*University of Colorado, Denver*

It is well-known that the discrete pair of spaces used in solving the Darcy problem by finite elements must satisfy an inf-sup condition. This places fairly severe limitations on the combination of spaces that can be used in a method. In particular, the element using piecewise linear velocity and piecewise linear pressure (both discontinuous and continuous) interpolations lead to an ill-posed linear system of equations. We develop a method which allows for the use of this pair of spaces by using the Petrov-Galerkin Enrichment Method (PGEM) to achieve stabilizing terms arising from a multiscale decomposition of the solution. The underlying approach correlates the enriched method to some local stabilized projection methods making the latter consistent. Numerical results show optimal convergence in natural norms and validate theoretical results.

### **A FRAMEWORK FOR PATTERN RECOGNITION IN MOLECULAR BIOLOGY DATA**

**Ryan Kennedy**  
*University of Colorado, Boulder*

In many biological systems such as gene regulatory networks, RNA splicing, and RNA polyadenylation, the individual parts of the system can interact in complex, nonlinear ways. As large numbers of biological sequences are accumulated, it is necessary to find methods that can uncover these complex relationships. Toward this goal, we present a framework of machine-learning techniques that can be used to identify patterns in

data taken from systems in molecular biology. These techniques are able to identify interactions relating to differences within individual sequences - including covariation between positions - as well as interactions between the sequences themselves. We demonstrate the use of these methods in looking for patterns related to the transcription factor p53, a protein known to have an effect on tumor suppression.

## **AFTERNOON SESSION III**

### *Modeling Contest, Problem B* **ESTIMATING THE IMPACT OF WIRELESS COMMUNICATIONS GROWTH ON ENERGY CONSUMPTION AND THE ENVIRONMENT**

**Yongli Chen, Tim Lewkow**  
*University of Colorado, Colo. Springs*

## *University of Colorado, Denver*

Amid the fears of global warming, ozone depletion, climate change, and global security, the general acceptance among the public is that something needs to change in our energy usage habits. Industry and low-efficiency automobiles are obvious culprits, but what other ways can we cut back on oil usage before it is too late? Some potential culprits worth examining are the many electronics we have come to take for granted, including cell phones. In the past decade, cell phones have evolved from a luxury to necessity. But what effect, if any, has the change had on our rate of oil consumption? We attempt to model the current telecommunications situation, and in the process, take a closer look at some seemingly innocent electrical devices, and ask a few What if's?

We used two models to gain a grasp of the current state of the telecommunications industry { an iterative C++ program and a Linear Programming (LP) model. The C++ program treated the situation on several different levels, including the household, city, region, and national levels. The program randomized individuals and households around given sample mean (from research) to reflect the probabilistic nature of the problem, and kept track of quantities such as each individual's cell phone's battery life, as well as when and how often a cell phone is charged.

The second model, the Linear Programming model, examined closer what constraints would prevent an optimal solution simply being an extreme situation with either 100% landlines or 100% cell phones. The two models' results closely mirrored each other, and both models agreed that neither extreme (that is, 100% cell or 100% landlines) was optimal, and that a relatively cellular-phone-heavy mixture was highly efficient.

Using the iterative C++ program, it was found that an optimal combination, both by energy efficiency and power-grid usage stability, was the use of 85.4% cellular phones and 14.6% landlines. When the simulation was run with these optimal parameters, a total energy savings over the In-

form stationary distribution may be found. Furthermore, the parameters for this system may be obtained empirically: how often cars arrive at an entrance (external arrival rate), how quickly is

while the other cylinder runs for a certain period of time. We seek to create and compare analytic, numerical, and experimental realizations of chaotic advection.

**A SYSTEMATIC CIRCUIT  
APPROACH TO MODELING  
NEURONS WITH AN ION PUMP**

**Tyler Takeshita**

*University of Northern Colorado*

Motivated by Professor Bo Deng's work, a systematic circuit approach to modeling neurons with an ion pump is presented. Like Dr. Deng, the voltage-gated current channels of a neuron are modeled as conductors, the diffusion-induced current channels are modeled as negative resistors, and the one-way ion pumps are modeled as one-way inductors. This model differs from the well-known Hodgkin-Huxley model because it splits the active and the passive branches of each ion species whereas the HH approach combines the electromagnetic, diffusive, and pump channels of each ion into one conductance channel. Our model maintains several of the known properties of HH models along with being rich in many new dynamical structures including chaotic behavior.