

Tenth Annual Front Range  
Applied Mathematics Student Conference  
March 1, 2014

**Registration: 8:30 - 9:00**

**Morning Session I - Room NC1323**

**9:00 - 11:35**

|               |  |  |
|---------------|--|--|
| 9:00 - 9:20   | Thomas Cabell-Kluch<br><i>Colorado School of Mines</i>         | Reconstruction of refractive index for a class of heterogeneous medium in atmospheric sciences         |
| 9:25 - 9:45   | David Nieves<br><i>University of Colorado, Boulder</i>         | Statistical classification of flow morphology in rapidly rotating Rayleigh-Benard convection           |
| 9:50 - 10:10  | Tony E. Wong<br><i>University of Colorado, Boulder</i>         | A model-based evaluation of the efficacy of evapotranspiration partitioning from stable water isotopes |
| 10:10 - 10:25 | 15 Minute Break  |  |
| 10:25 - 10:45 | Brent Davis<br><i>Colorado State University</i>                | New Computational Tools for Analyzing Models of Phylogenetic Trees                                     |
| 10:50 - 11:10 | George Borleske<br><i>Colorado State University</i>            | Comparing Roughness Metrics with Morphometric Roughness Lengths for a Snowpack Surface                 |
| 11:15 - 11:35 | Volodymyr Kondratenko<br><i>University of Colorado, Denver</i> | Assimilation of the ice perimeter data using the ice spread model SFIRE coupled with the WRF model     |

**Morning Session II - Room NC1324**

**9:00 - 11:35**

|               |  |   |
|---------------|--|---|
| 9:00 - 9:20   | Deborah Shutt<br><i>Colorado School of Mines</i>         | Models of in-host Population Dynamics of the HIV-1 Virus and CD4 <sup>+</sup> T cells |
| 9:25 - 9:45   | Farrah Sadre-Marandi<br><i>Colorado State University</i> | Geometric Aspects of Viral Capsids  |
| 9:50 - 10:10  | Inom Mirzaev<br><i>University of Colorado, Boulder</i>   | Laplacian Dynamics of Biochemical Synthesis and Degradation                           |
| 10:10 - 10:25 | 15 Minute Break  |   |

|               |   |  |
|---------------|---|--|
| 10:25 - 10:45 | Stephen Kissler<br><i>University of Colorado, Boulder</i>   | Determination of personalized diabetes treatment plans using a two-delay model |
| 10:50 - 11:10 | Victoria Gershuny<br><i>University of Colorado, Boulder</i> | Changes in the lateral diffusion of surface molecules                          |
| 11:15 - 11:35 | Mike Mikucki<br><i>Colorado State University</i>            | Modeling membrane dynamics with surface harmonics                              |

## Morning Session III - Room NC1325

### 9:00 - 11:35

|               |  |   |
|---------------|--|---|
| 9:00 - 9:20   | Ruth A. Martin<br><i>University of Colorado, Boulder</i>   | Three-wave resonant interactions  |
| 9:25 - 9:45   | Ahmad Alyoubi<br><i>Colorado School of Mines</i>           | Parallel-in-Time HPC Algorithms for Fractional Space-Time Models  |
| 9:50 - 10:10  | Brad Dworzak<br><i>Colorado School of Mines</i>            | A non-polynomial FEM for simulation of scattering from multiple particle configurations   |
| 10:10 - 10:25 | 15 Minute Break  |   |
| 10:25 - 10:45 | Justin Praast<br><i>Colorado School of Mines</i>           | A finite-difference time-domain method for first-order Maxwell systems  |
| 10:50 - 11:10 | Nathan D. Monnig<br><i>University of Colorado, Boulder</i> | Inverting Non-Linear Dimensionality Reduction with Scale-Free Radial Basis Interpolation  |
| 11:15 - 11:35 | Bradley Martin<br><i>University of Colorado, Boulder</i>   | Accurate seismic wave simulation through radial basis function-derived finite differences (RBF-FD): preliminary results from a new 3rd-order numerical method |

### Lunch: 11:45 - 12:25

## Plenary Address: 12:30 - 1:30, Room NC1539

Dr. Stephan Sain, *Computational and Information Systems Laboratory, Geophysical Statistics Program - NCAR, Boulder*

Solving the Earth's Equations: Mathematics and Statistics at the National Center for Atmospheric Research

### Group Photographs at 1:30

## Afternoon Session I - Room NC1323

### 1:45 - 3:50

|             |  |   |
|-------------|--|---|
| 1:45 - 2:05 | Melody Dodd<br><i>Colorado State University</i>        | A fast implementation of the D-bar algorithm for electrical impedance tomography        |
| 2:10 - 2:30 | Yan Chen<br><i>University of Colorado, Boulder</i>     | Convergent Series Solution to 1D Schrodinger Equation                                   |
| 2:35 - 2:55 | Tim Hodges<br><i>Colorado State University</i>         | Computing Syzygies of Homogeneous Polynomials Using Linear Algebra                      |
| 3:00 - 3:20 | Rees McNally<br><i>University of Colorado, Boulder</i> | Gravity Gradiometry: A Novel Application for Compressed Sensing                         |
| 3:25 - 3:45 | Charles Morgenstern<br><i>Colorado School of Mines</i> | Coupled Finite and Boundary Element Methods for Wave Propagation in Inhomogeneous Media |

## Afternoon Session II - Room NC1324

### 1:45 - 3:50

|             |  |   |
|-------------|--|---|
| 1:45 - 2:05 | Axel Brandt<br><i>University of Colorado, Denver</i>       | Turan Numbers of Expanded Hypergraphs   |
| 2:10 - 2:30 | Rachel Neville<br><i>Colorado State University</i>         | A Pattern in Chaos: Persistent Homology of the Logistic Map                                     |
| 2:35 - 2:55 | Warren M. Lord<br><i>University of Colorado, Boulder</i>   | A singular value decomposition approach to connecting topology and dynamics on complex networks |
| 3:00 - 3:20 | Yogesh S. Virkar<br><i>University of Colorado, Boulder</i> | Hamiltonian Mean Field Model: Effect of network structure on synchronization dynamics           |
| 3:25 - 3:45 | Tim Marrinan<br><i>Colorado State University</i>           | Detecting Dampened Signals in Image Subspaces Using the Flag Mean                               |

## Afternoon Session III - Room NC1325

1:45 - 3:50

|             |   |   |
|-------------|---|---|
| 1:45 - 2:05 | Prosper Torsu<br><i>University of Wyoming</i>               | A Truncation Method for Elliptic Boundary Value Problems: An Application to Monte Carlo Simulations |
| 2:10 - 2:30 | Evan Kwiatkowski<br><i>University of Colorado, Denver</i>   | Convergence of the Unbiased Square Root Kalman Filter in the Large Ensemble Limit                   |
| 2:35 - 2:55 | Jennifer Diemunsch<br><i>University of Colorado, Denver</i> | Robber Locating Strategy for Trees  |
| 3:00 - 3:20 | V. Ganesh Karapakula<br><i>Colorado College</i>             | Optimizing Class Schedules on the Block Plan  |
| 3:25 - 3:45 | James Syme<br><i>University of Colorado, Denver</i>         | A Mathematical Model of Political Borders   |

# Plenary Speaker

12:30 - 1:30, NC1539

*Solving the Earth's Equations: Mathematics and Statistics at*

# MORNING SESSION I

## RECONSTRUCTION OF REFRACTIVE INDEX FOR A CLASS OF HETEROGENEOUS MEDIUM IN ATMOSPHERIC SCIENCES

Thomas Cabell-Kluch  
*Colorado School of Mines*

The ability to accurately correct for the effects of strong atmospheric turbulence on light has been a major area of research in astronomy and adaptive optics. Atmospheric turbulence induces an optical region with inhomogeneous refractive index. This research's main focus is to efficiently reconstruct the refractive index using observed/simulated optical phase aberrations. A standard reconstruction method is based on the tomography associated with these aberrations. In this work, assuming the turbulent atmosphere is of Kolmogorov type, we extend the range of this reconstruction. We demonstrate the approach for simulated data from an astronomy model.

## STATISTICAL CLASSIFICATION OF FLOW MORPHOLOGY IN RAPIDLY ROTATING RAYLEIGH-BENARD CONVECTION

David Nieves  
Collaborators: Antonio Rubio and Keith Julien  
*University of Colorado, Boulder*

In rapidly rotating convection four flow regimes with distinct characteristics have been identified as a function of the asymptotically reduced Rayleigh number  $RaE^{4-3}$  (where  $E$  is the non-dimensional Ekman number) and Prandtl number. In each regime the flow organizes, with varying intensity, into coherent vertical structures. The identified morphologies, in order of increasing  $RaE^{4-3}$ , consist of the cellular regime, the convective Taylor column (CTC) regime, the plume regime, and a regime characterized by geostrophic turbulence. We use experimentally accessible statistical measures to distinguish between flow morphologies in rapidly rotating Rayleigh-Benard convection (RRBC). Namely, the utilization of auto- and cross-correlations of

temporal and spatial signals that synthesize experimental data obtained from thermistor measurements or particle image velocimetry (PIV). We show how these statistics can be employed for laboratory experiments to (i) identify transitions in the flow morphology, (ii) capture the radial profiles of coherent structures, and (iii) extract transport properties of these structures.

## A MODEL-BASED EVALUATION OF THE EFFICACY OF EVAPOTRANSPIRATION PARTITIONING FROM STABLE WATER ISOTOPES

Tony E. Wong<sup>1</sup>  
Collaborators: Max Berkelhammer<sup>2</sup> and David Noone<sup>1</sup>  
*University of Colorado, Boulder*<sup>1</sup>  
*University of Illinois*<sup>2</sup>

The partitioning of evapotranspiration (ET) into ground evaporation and plant transpiration in land surface models is crucial for understanding the surface energy balance and a key feature of the water cycle. (CONVEW8t9091 Tf 4-3.959 -00Td (one)]0.909

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and surface and soil water isotope ratios, showing that soil water isotopic information holds the most promise for further constraining ecosystem process models.

**NEW COMPUTATIONAL TOOLS FOR  
ANALYZING MODELS OF  
PHYLOGENETIC TREES**

Brent Davis<sup>1</sup>

Collaborator: Joseph Rusinko<sup>2</sup>

*Colorado State University*<sup>1</sup>

*Winthrop University*<sup>2</sup>

Computational Phylogenetics studies compu-

## MORNING SESSION II

### MODELS OF IN-HOST POPULATION DYNAMICS OF THE HIV-1 VIRUS AND CD4<sup>+</sup>T CELLS

Deborah Shutt  
*Colorado School of Mines*

We will describe and analyze a mathematical model capable of predicting the time course of HIV infection within the human body. Upon entering a host, HIV infects CD4<sup>+</sup>T cells, a key component of the immune system, in order to replicate. Thus, the virus will inhibit the immunological function of the T cell. During the replication process, mistakes can occur and a new mutated strain of HIV can develop. This mutated strain will then possess a new structure which may allow it to escape the body's immune response or resist the effects of anti-retroviral drugs. Uninfected CD4<sup>+</sup>T cells are left to work against the infection but the presence of HIV in the body will eventually lead to a depletion of these cells and finally the development of Acquired Immune Deficiency Syndrome (AIDS). Using biological data such as infection rates, production rates of virions from infected T-cells, and mutation rates, we construct a model which examines the in-host population dynamics of the virus. The result is a nonlinear seven component ODE model and we examine the qualitative behavior of the various populations through mathematical analysis and computational approximations.

### GEOMETRIC ASPECTS OF VIRAL CAPSIDS

Farrah Sadre-Marandi  
*Colorado State University*

The assembly of proteins on a virus shell, called the capsid, are marvels of molecular architecture. We can observe symmetry at various levels and the important roles of hexagons and pentagons in viral capsid structures. This talk starts from icosahedral virus but focuses on the features of the HIV-1 conical core. We examine the critical role of pentamers in the HIV-1 core

and present novel results of curvature calculations.

### LAPLACIAN DYNAMICS OF BIOCHEMICAL SYNTHESIS AND DEGRADATION

Inom Mirzaev  
*University of Colorado, Boulder*

In recent years, many researchers have devoted their efforts to developing a systems-level understanding of biochemical reaction networks. In particular, the study of these chemical reaction networks (CRNs) using their associated graph structure has attracted considerable attention. In our previous work, we have developed a framework for finding steady states of a biochemical reactions. Specifically, we associated each biochemical network with a labeled, directed graph,  $G$ , and with the corresponding Laplacian matrix. If there is an edge from vertex  $j$  to vertex  $i$ , we label it with  $e_{ij} > 0$ , and with  $e_{ij} = 0$  if there is no such edge. The Laplacian matrix of given digraph  $G$  is then defined as

$$(L(G))_{ij} = \begin{cases} e_{ij} & \text{if } i \neq j \\ \sum_{m \neq j} e_{mj} & \text{if } i = j \end{cases}$$



any combination of synthesis and degradation edges. The dynamics can now be described by the following system of linear ordinary differential equations (ODEs):

$$\frac{dx}{dt} = L(G)x - Dx + S;x$$

to this energy minimization problem have been obtained in various ways. Solving the Euler-Lagrange equations directly over the entire membrane surface has yielded only axisymmetric conformations. For nonaxisymmetric conformations, the solution can be found by minimizing the energy over a smaller subspace. A triangulated approximation of the membrane surface, as used by finite element methods, can be computationally

this preliminary research is on efficient and accurate simulation of large number of realizations of the model arising from sampling of the random attributes. In particular, we apply a high-order non-polynomial finite element method (FEM) for each realization. We demonstrate the approach for a class of model configurations.

A radial basis function approximation method

## **A FINITE-DIFFERENCE TIME-DOMAIN METHOD FOR FIRST ORDER MAXWELL SYSTEMS**

**Justin Praast**  
*Colorado School of Mines*

We consider a finite-difference method in space and time for efficiently simulating non-time-harmonic first-order Maxwell systems in  $R^n$ ;  $n = 1;2;3$ . An industrial standard approach to approximate the time-dependent system for unknown electric and magnetic fields is the explicit finite-difference time-domain (FDTD) method on the Yee cell grid. The Yee cell facilitates the expression of essential physical relationships of electromagnetic fields and the approach is based on the idea of staggered grid evaluation.

In this presentation, we discuss the Yee cell based algorithm, its stability constraints given by the CFL condition, dispersion and dissipation relations and its relation to the CFL condition, material boundary conditions, and artificial domain boundary conditions. We demonstrate preliminary FDTD simulation results of a monochromatic and a broadband pulse input in one and two dimensions and consider a parallel computing implementation of the FDTD algorithm.

## **INVERTING NON-LINEAR DIMENSIONALITY REDUCTION WITH SCALE-FREE RADIAL BASIS FUNCTIONS**

**Nathan D. Monnig**  
*University of Colorado, Boulder*

A numerical method is proposed to approximate the inverse of a general bi-Lipschitz nonlinear dimensionality reduction mapping, where the forward and consequently the inverse mappings are only explicitly defined on a discrete dataset.

# AFTERNOON SESSION I

## A FAST IMPLEMENTATION OF THE D-BAR ALGORITHM FOR ELECTRICAL IMPEDANCE TOMOGRAPHY

Melody Dodd  
*Colorado State University*

Electrical impedance tomography is an imaging technique in which applied boundary currents and resulting boundary voltages are used to reconstruct the conductivity distribution within a body. D-bar methods are a class of direct (non-iterative) reconstruction algorithms that make use of complex geometrical optics (CGO) solutions to PDEs known as D-bar, or  $\bar{\partial}$ , equations. D-bar equations are of the form  $\bar{\partial}u = f$ ; where  $f$  may depend on  $u$ , and the  $\bar{\partial}$  operator is defined by  $\bar{\partial} = 0.5(\partial_x + i\partial_y)$ . The common threads in these methods are (1) a direct relationship between the CGO solutions and the unknown conductivity, (2) a nonlinear Fourier transform, also known as the scattering transform, providing a link between the data and the CGO solution, and (3) a D-bar equation to be solved for the CGO solutions with respect to a complex-frequency variable.

## CONVERGENT SERIES SOLUTION TO 1D SCHRODINGER EQUATION

Yan Chen  
*University of Colorado, Boulder*

The  $N^{th}$  partial sum of an infinite power series has the form  $\sum_{n=1}^N c_n x^n$ , where  $x$  is a parameter and  $c_n$  are known numbers. The infinite series is *convergent* if this sum has a finite limit as  $N \rightarrow \infty$ , with  $x$  fixed. The series is *asymptotic* if the error incurred by truncating the series after  $N$  terms is  $O(x^{-(N+1)})$  as  $x \rightarrow 0$ . The two concepts, which involve two different limits, are independent - a given series might have both properties, or one, or neither.

Divergent asymptotic series can be quite useful: the well known formula of Stirling (1730) for  $\ln(m!)$  for large  $m$  is a divergent series (in powers of  $\frac{1}{m}$ ) for all finite  $m$ , but Stirling calculated

$\log_{10}(1000!)$  to ten decimal places using only a few terms in his series. (see Copson, 1967).

Divergent asymptotic series are most useful when the parameter  $x$  is small but not zero. In this situation, with  $x$  fixed, one finds that the error gets smaller as one keeps more terms in the series until a specific point, after which the error begins to grow. In this way, a divergent asymptotic series can approximate a function for small, fixed  $x$  up to some optimal accuracy, but no better. Dingle (1973) began development of a method to create a follow-on series, which begins at the point of optimal accuracy of the original series, and can be used to improve the accuracy further, but again with a minimal error that cannot be reduced by further use of the two series. Berry & Howls (1990) developed Dingle's idea further, using a sequence of divergent series, with each series improving on the accuracy of the preceding one. Again, they eventually came to a (much smaller) minimal error that they could not reduce. They named this sequence of increasingly accurate asymptotic series *hyperasymptotics*. They demonstrated their approach by developing hyperasymptotic series, valid for large positive  $z$ , for the Airy function,  $\text{Ai}(z)$ , and showed that they could evaluate  $\text{Ai}(z)$  with 20 decimal places of accuracy, even down to  $z = 5.241$ .

## COMPUTING SYZYGIES OF HOMOGENEOUS POLYNOMIALS USING LINEAR ALGEBRA

Tim Hodges  
*Colorado State University*

Polynomial systems arise in science and engineering frequently. Using Algebraic Geometry we can solve these polynomial systems and find relations for the polynomials. In particular, given a ideal generated by polynomials  $f_1, \dots, f_n$  in  $C[x_1, \dots, x_m]$  a syzygy is a  $n$ -tuple  $(s_1, \dots, s_n)$ ,  $s_i \in C[x_1, \dots, x_m]$  such that  $\sum_{i=1}^n s_i f_i = 0$ . That

time Gröbner bases have been computationally impractical as the number of variables and number of polynomials increase. The aim of this research is to give a way to compute syzygies without the need for Gröbner bases and possibly still give the same information as Gröbner bases. The approach is to use the monomial structure of each polynomial in our generating set to build syzygies using nullspace computations.

## **GRAVITY GRADIOMETRY: A NOVEL APPLICATION FOR COMPRESSED SENSING**

**Rees McNally<sup>1</sup>**

**Collaborators: Stephen Libby<sup>2</sup> and David Chambers<sup>2</sup>**

*University of Colorado, Boulder<sup>1</sup>*

*Lawrence Livermore National Lab<sup>2</sup>*

During the past decade, compressed sensing (CS) has proven to be extremely useful for sparse signal reconstruction. For this work the method of CS is applied to fast mass distribution determination based on cold atom gravity gradiome-

# AFTERNOON SESSION II

## TURAN NUMBERS OF EXPANDED HYPERGRAPHS

Axel Brandt<sup>1</sup>

Collaborator: Tao Jiang<sup>2</sup>

*University of Colorado, Denver*<sup>1</sup>

*Miami University*<sup>2</sup>

A graph  $G$

distributions on random networks, but do not perform as well when structures that are common to real world networks, such as commu-

to be represented with a single geometric object, and forces common information to the front of the flag. This property acts like a subspace median, and can be beneficial for identifying weak or dampened signals common to the the collections of images. We present preliminary results on how these image signals can be detected and classified using a flag representation for a collection of these subspaces.

## AFTERNOON SESSION III

### A TRUNCATION METHOD FOR ELLIPTIC BOUNDARY VALUE PROBLEMS: AN APPLICATION TO MONTE CARLO SIMULATIONS

Prosper Torsu  
*University of Wyoming*

Truncation methods have been used widely to estimate solutions of complicated mathematical problems. One particular type that has been implemented represents the elliptic coefficient as an infinite series of some bounded function. The approximate solution is represented as a truncation of an infinite series. The structure of the method is such that these solutions are obtained successively. They are then summed up to approximate the true solution. A consequence of this method is that a higher-order solution depends on all lower-order ones. Hence, creating data for a fixed higher-order problem forces us to invert a system as many times as the order of the problem.

In this study, we propose an alternative method that minimizes the computational burden. The most important advantage of the proposed method is that irrespective of the order of approximation, we only need to invert the system twice making the whole calculation much more efficient. A convergence analysis of the perturbation method will be given.

The proposed method is directly applicable in the framework Monte-Carlo simulation. Monte-Carlo simulation can be very expensive especially when the coefficients relatively complicated. With the proposed method, the system inversion is

very much minimized thereby making the simulation more efficient. The performance of the method within the Monte-Carlo simulation will be shown, in particular as it compares to traditional Monte-Carlo simulation.

### CONVERGENCE OF THE UNBIASED SQUARE ROOT KALMAN FILTER IN THE LARGE ENSEMBLE LIMIT

Evan Kwiatkowski  
Collaborator: Jan Mandel  
*University of Colorado, Denver*

Ensemble filters implement sequential Bayesian estimation by representing the probability distribution by an ensemble mean and covariance. Unbiased square root ensemble filters use deterministic algorithms to produce an analysis (posterior) ensemble with prescribed mean and covariance, consistent with the Kalman update. This includes several filters used in practice, such as the Ensemble Transform Kalman Filter (ETKF), the Ensemble Adjustment Kalman Filter (EAKF), and a filter by Whitaker and



her turn, and is told the distance to the robber. The cop wins when she can determine the location of the robber, and the robber wins if the cop can never determine the robber's location. This talk discusses an algorithm that meets the known bound for the number of turns needed to locate a robber on a tree.

## **OPTIMIZING CLASS SCHEDULES ON THE BLOCK PLAN**

**V. Ganesh Karapakula**  
*Colorado College*

Colorado College operates on the Block Plan, which is an academic schedule consisting of eight blocks per academic year. A block lasts for three and a half weeks; the students take one class at a time every block. Every academic department at the college makes an annual schedule of classes under various department-wide and professor-specific constraints. Department-wide constraints include limitations on the conjunction of some courses and on the number and the timings of offerings of each kind of course. Professor-specific constraints include restrictions on the number and the sorts of courses and the blocks in which a professor can teach. Literature dealing with optimization of class schedules does not adequately consider Block Plan-type systems. This talk tackles optimization of class schedules in settings such as the Block Plan using binary integer linear programming; solutions to similar optimization problems will also be explored in this talk.

## **A MATHEMATICAL MODEL OF POLITICAL BORDERS**

**James Syme**  
*University of Colorado, Denver*

Cities and metropolitan areas act as the centers of national cultural, linguistic, governmental, and economic exchange, while also acting as portals for international immigration and foreign relations. I hypothesize that, due to relative cultural dispersion from city centers, pre-eminent world and primate cities can act as generating nodes for the geographic expanse of nation states. I present a 3 phase model of the

geographic territorial expanse of nation states: (1) use bounded Voronoi tessellations generated with longitude/latitude coordinates of cities as central nodes to create regions of urban influence (RUIs); (2) generate countries using RUIs, an alliance matrix, and simulated border segments; and (3) use dynamics to update the alliance matrix and generate new states. A dynamical model is presented to demonstrate the ability of the model with the intent of encouraging social scientists to apply their own models.