

APPM 1340

Exam 2

Fall 2022

|            |                  |             |
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| Name       |                  |             |
| Instructor | Richard McNamara | Section 150 |

This exam is worth 100 points and has **4 problems**.

**Make sure all of your work is written in the blank spaces provided.** If your solutions do not fit, there is additional space at the end of the test. Be sure to **make a note** indicating the page number where the work is continued or it will **not** be graded.

**Show all work and simplify your answers.** Name any theorem that you use. Answers with no justification will receive no points unless the problem explicitly states otherwise.

Notes, papers, calculators, cell phones, and other electronic devices are not permitted.

L'Hopital's Rule and dominance of powers arguments can **NOT** be used to evaluate limits on this exam

1. (20 pts) Parts (a) and (b) are not related.

(a) For  $f(x) = \frac{1}{x^2}$  and  $g(x) = \frac{1}{x+2}$ , identify the composite function  $(f \circ g)(x)$  and its domain. Express the domain in interval form.

(b) The graph below depicts a function of the form  $y = h(x) = a \sin (bx) + c$

2. (30 pts) Evaluate the following limits. Support your answers by stating theorems, definitions, or other key properties that are used.

(a)  $\lim_{x \neq 0} \frac{\tan x \sin(2x)}{x^2}$

$$(b) \lim_{x \rightarrow 9} \frac{\sqrt{x-5} - 2}{x-9}$$

(c)  $\lim_{x \neq 0} x^8 \sin \frac{1}{x^3}$

3. (30 pts) Consider the rational function  $r(x) = \frac{x^2 - 5x + 4}{2x^2 - 8x + 6}$ .

- (a) Identify all values of  $x$  at which  $r(x)$  is discontinuous. At each such  $x$  value, explain why the function is discontinuous there.

(b) Identify the type of discontinuity associated with each  $x$  value identified in part (a). Support those classifications by evaluating the appropriate limits.



(c) Find the equation of each vertical asymptote of  $y = r(x)$ , if any exist. Support your answer in terms of the limits you evaluated in part (b).

(d) Find the equation of each horizontal asymptote of  $y = r(x)$ , if any exist. Support your answer by evaluating the appropriate limits.

4. (20 pts) Parts (a) and (b) are not related.

(a) For what value of  $b$  is the following function  $u(x)$  continuous at  $x = 3$ ? Support your answer using the definition of continuity, which includes evaluating the appropriate limits.

$$u(x) = \begin{cases} \frac{x^2 - 9}{x - 3} & ; x < 3 \\ 5x + b & ; x \geq 3 \end{cases}$$

- (b) The Intermediate Value Theorem can **NOT** be used to guarantee that  $v(x) = \frac{2}{x} + \frac{1}{x+2} = 0$  for a value of  $x$  on the interval  $(-1; 2)$ . Explain which condition for applying the theorem is not satisfied in this case.

END OF TEST

Your Initials \_\_\_\_\_

ADDITIONAL BLANK SPACE

If you write a solution here, please clearly indicate the problem number.