

**Exam 3****Instructions:**

- This exam has four problems on pages numbered 1 through 9. Make sure you have all pages.
- Write your name and section number at the top of each page.
- Show all work and simplify your answers, except where the instructions tell you to leave your answer unsimplified.
- Name any theorem that you use and explain how it is used.
- Answers with no justification will receive no points unless the problem explicitly states otherwise.
- Notes, your text and other books, calculators, cell phones, and other electronic devices are not permitted, except as needed to upload your work.
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1. (40 pts)

(a) Evaluate the integral if it exists.

i.  $\int \frac{3x + 9}{x^2 + 6x} dx$

ii.  $\int_{\frac{1}{2}}^1 2 \sin(v) \cos(v) dv$

iii.  $\int_3^3 (2x^4 + 3x) dx$

(b)

$$\begin{aligned} \int_{\frac{3}{4}}^{\frac{1}{4}} \cos(2x) dx &= -\frac{1}{2} \sin(2x) \Big|_{\frac{3}{4}}^{\frac{1}{4}} \\ &= -\frac{1}{2} [\sin(\frac{1}{2}) - \sin(\frac{3}{2})] \\ &= \frac{1}{2} \end{aligned}$$

(c) Since  $g$  is odd  $\int_{-5}^5 g(x) dx = 0$ .

$$\begin{aligned} \int_{-5}^{-2} g(x) dx + \int_{-2}^0 g(x) dx + \int_0^5 g(x) dx &= 0 \\ \int_{-5}^{-2} g(x) dx - \int_0^2 g(x) dx + \int_0^5 g(x) dx &= 0 \\ \int_{-5}^{-2} g(x) dx + \int_0^3 g(x) dx &= 0 \\ \int_{-5}^{-2} g(x) dx &= -\int_0^3 g(x) dx = -14 \end{aligned}$$

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2. (12 pts)

A fence is to be built to enclose a rectangular area of 250 square feet. The fence along three sides is to be made of material that costs 6 dollars per foot. The material for the fourth side will cost 10 dollars per foot. Find the dimensions of the enclosure that minimize the cost of fencing material.

**Solution:**

Let  $x$  be the width and  $y$  be the length of the enclosed area, and suppose that one of the sides of length  $y$  costs \$10. The two equations we have are:

$$A = xy = 250 \tag{1}$$

$$C = 6(2x)$$

3. (24 pts)

(a) Suppose an object moves with velocity  $v(t) = 2t^2 - 12t + 16$  km/hr along a straight road.

i. Determine the displacement of the object on the time interval  $[1,3]$ .

ii. Determine the distance traveled on the time interval  $[1,3]$ .

(b) Apply Newton's method to the equation  $x^3 + x - 5 = 0$ : Use an initial guess of  $x_0 = 1$  and find  $x_1$ : (Find only  $x_1$ .)

**Solution:**  $x +$

(b)  $f(x) = x^3 + x - 5$  and  $f'(x) = 3x^2 + 1$ .

$$\begin{aligned}x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\&= 1 - \frac{1 + 1 - 5}{3 + 1} \\&= 1 + \frac{3}{4} \\&= \frac{7}{4}\end{aligned}$$

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4. (24 pts)

- (a) Evaluate the Riemann sum for  $f(x) = x^2 - 3$  taking the sample points to be right endpoints,  $a = -4$ ,  $b = 2$  and  $n = 6$ .
- (b) Express the integral  $\int_{-4}^2 (x^2 - 3) dx$  as a limit of Riemann sums. You are not required to fully simplify this expression.
- (c) Evaluate the expression that you gave in (b). Show all steps to find the limit of the Riemann sums.

**Solution:**

(a) With  $a = -4$ ;  $b = 2$ ;  $n = 6$ ,  $\Delta x = \frac{2 - (-4)}{6} = 1$ . We make a table:

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Section \_\_\_\_\_

Name