

1. (34 points) Find the requested information. The problems are unrelated.

(a) Evaluate  $\int \frac{\tan^{-1}(x)}{x^2} dx$  (Hint: Start with IBP)

(b) Find  $y$  as a function of  $x$  given that  $\frac{dy}{dx} = 2x^p \sqrt{1-y^2}$  and  $y(0) = 1$

(c) Find the sum of the series  $\frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots$

(d) For what values of  $x$  does the series  $\sum_{n=0}^{\infty} \sin^2(x) \cos^{2n}(x)$  converge? Find the sum for those values of  $x$

2. (16 points) Decide whether the following quantities are convergent or divergent. Explain your reasoning and any test you use.

(a) The sequence given by  $a_n = 1 - \frac{\ln(3)^n}{n}$ , for  $n = 1; 2; \dots$

(b)  $\int_1^{\infty} \frac{1}{x^2} \sqrt{1 + \frac{3}{x^3}} dx$

3. (12 points) Consider the series  $\sum_{k=1}^{\infty} a_k$ . Suppose the  $n$ th partial sum of the series is  $2 - \frac{2}{n+1}$ .

(a) What is  $a_3$ ?

(b) Find a simple formula for  $a_n$

(c) What does  $\sum_{k=1}^{\infty} a_k$  converge to?

(d) What is the sum of the series  $\sum_{k=1}^{\infty} k a_k$ ?

4. (25 points) Recall  $\cosh(x) = \frac{e^x + e^{-x}}{2}$ .

(a) Find the MacLaurin series of the hyperbolic cosine function.

(b) Find the interval of convergence for the power series from part (a).

(c) Find  $T_3(x)$ , the Taylor polynomial of order 3, of the hyperbolic cosine centered at  $x=1$ . Use the Taylor Remainder formula to find an upper bound for the absolute error if  $T_3(x)$  is used to approximate  $\cosh(1)$ .

(d) Use the MacLaurin series (no l'Hôpital!) to evaluate the following limit:

$$\lim_{x \rightarrow 0} \frac{\cosh(x) - 1 - \frac{x^2}{2}}{x^4}$$

5. (18 points) Suppose  $f(x)$  equals the power series  $\sum_{n=2}^{\infty} \frac{(n+1)(x+b)^n}{c^{2n}}$ , where  $b$  and  $c$  are constants, and the series has an interval of convergence of  $|x-b| < 2$ .

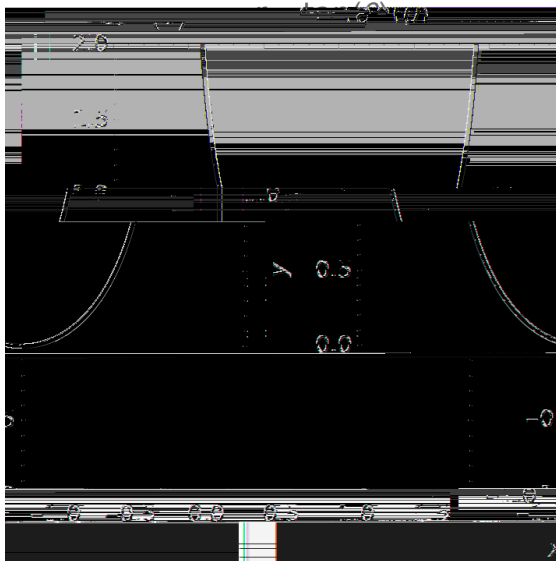
(a) Find the center and radius of convergence of the series.

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(b) Evaluate  $\int g(x) dx$  as a power series.

(c) Given the interval of convergence, find possible values for  $\theta$ . Justify your answer using appropriate test(s).

6. (25 points) For this problem, let  $f(x) = \tan^{-1}(x)$  for  $-2 < x < 2$ . The polar graph (in the  $xy$ -plane) is given below. Answer the following questions.



(a) Find an equation for the tangent line to the curve at  $(1, 0.5)$ .