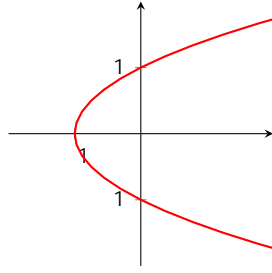


APPM 2350—Exam 1
Wednesday Sep 22nd, 6:30pm-8pm 2021

(c) Sketch.



(b) The missile will travel along the tangent line to the jet at the point $(2; 2; \frac{4}{3})$.

To find the equation of this tangent line, we need a point on the line and a vector in the direction of the missile.

Point on the tangent line: $(2; 2; \frac{4}{3})$

Vector in the direction of the missile = $\mathbf{r}'(2) = \langle 4; 8; \frac{8}{3} \rangle$ (using our parameterization from part (a)).

However, notice that the speed of $\mathbf{r}'(2)$ is not the same as the missile's speed.

The missile's velocity vector = $(12) \frac{\mathbf{r}'(2)}{\|\mathbf{r}'(2)\|} = 12 \frac{\langle 4; 8; \frac{8}{3} \rangle}{3} = \langle 16; 32; 32 \rangle$

Thus, the missile will travel along the line given below for $t \geq 0$:

$$x = 2 + 16t$$

$$y = 2 + 32t$$

$$z = \frac{4}{3} + 32t$$

After traveling along this line for 5 seconds, the missile will be at the point:

$$x = 2 + 16(5) = 82$$

$$y = 2 + 32(5) = 162$$

$$z = \frac{4}{3} + 32(5) = \frac{164}{3}$$

$$\Rightarrow (x; y; z) = (82; 162; \frac{164}{3})$$

Problem 4 (20 points)

A particle travels along a curve parameterized by

$$\mathbf{r}(t) = \langle 4t; \cos(3t); \sin(3t) \rangle; \quad 0 \leq t \leq \pi$$

where t is time.

(a) At what coordinates $(x; y; z)$, if any, is the particle's unit normal vector, $\mathbf{N}(t)$ parallel to the following plane? Explain/justify your answer.

$$\frac{3}{2}x + y - \frac{1}{3}z = \frac{2}{3}$$

(b) At what time(s), if any, is the curvature of the particle's path equal to $\frac{1}{2}$? Explain/justify your answer.

SOLUTION:

(a)

$$\mathbf{r}'(t) = \langle 4; -3 \sin(3t); 3 \cos(3t) \rangle$$

$$\|\mathbf{r}'(t)\| = \sqrt{16 + 9 \sin^2(3t) + 9 \cos^2(3t)} = 5$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|} = \frac{1}{5} \langle 4; -3 \sin(3t); 3 \cos(3t) \rangle$$

$$\frac{d}{dt} \mathbf{T}(t) = \frac{1}{5} \langle 0; -9 \cos(3t); -9 \sin(3t) \rangle$$

$$\mathbf{N}(t) = \frac{\frac{d}{dt} \mathbf{T}(t)}{\|\frac{d}{dt} \mathbf{T}(t)\|} = \langle 0; \cos(3t); \sin(3t) \rangle$$

(d)

$$\begin{aligned} a_N' &= 0 \\ \frac{2c}{1+4(t-1)^2} &= 0 \\ \frac{8c(t-1)}{(1+4(t-1))^3} &= 0 \\ t &= 1 \quad (5 \text{ pts}) \end{aligned}$$

$a_N' > 0$ when $t < 1$ and $a_N' < 0$ when $t > 1$) $a_N(t=1) = 2c$ is a maximum. (2 pts)
(e) $a_N(t=1) = 2c = g$, therefore for the car not to roll over we need $c \geq \frac{g}{2}$