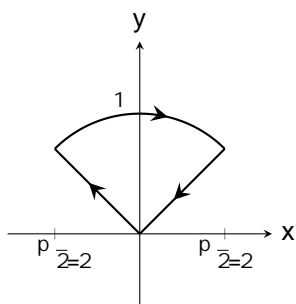


1. [2350/050823 (46 pts)] A wire is in the shape of the curve given by $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + t^2\mathbf{k}$; $1 \leq t \leq 2$
- (a) [9 pts] Does the wire intersect the plane that contains the points $(1, 2, 0)$, $(3, 0, 3)$, $(0, -1, 4)$? If so, find the point of intersection. If not, explain why not.
- (b) [5 pts] What is the curvature of the wire when $t = 1$?
- (c) [15 pts]

2. [2350/050823 (2006)] Consider the oriented curve C shown in the figure (the curved portion is an arc of the unit circle). Compute the circulation of \mathbf{f} on C where $\mathbf{f} = (6y + \sin x^2) \mathbf{i} + (4e^y + 3x^2) \mathbf{j}$.



SOLUTION :

Z

4. [2350/050823 (2020)] Compute $\int_C P dx + Q dy + R dz$ where $F = P\mathbf{i} + Q\mathbf{j} + R\mathbf{k} = y\mathbf{i} + x^2\mathbf{j} + z\mathbf{k}$ by evaluating an appropriate surface integral. C is the boundary of the portion of the plane $5z = 1$ in the first octant, oriented counterclockwise when viewed from above.

SOLUTION:

We use Stokes' Theorem.

$$\mathbf{r} = y\mathbf{i} + x^2\mathbf{j} + z\mathbf{k}$$

$$\mathbf{F} = y\mathbf{i} + x^2\mathbf{j} + z\mathbf{k}$$

The surface is $g(x; y; z) = x + y + 5z$ which we project onto the xy -plane giving R as the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$.

