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abstract

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$$\left| r^- - \sum_{m=1}^M w_m \right|$$

1. Let us assume that (4) holds. For any $\epsilon > 0$ and $t_0 \in \mathbb{R}$, we have

$$\left| \int_{\mathbb{R}} f(t) dt - h \sum_{n \in \mathbb{Z}} f(t_0 + nh) \right| \leq \epsilon \tag{6}$$

provided that the Fourier transform of f satisfies

$$|\hat{f}(\xi)| \leq c_1 e^{-q|\xi|}, \tag{7}$$

for some positive constants c_1, q and step size $h \leq \frac{1}{2c_1} (2c_1^{-1} + 1)$ or, alternatively,

$$|\hat{f}(\xi)| \leq \frac{c_2}{|\xi|^q}, \quad \text{for } |\xi| \geq R, \tag{8}$$

for some positive constants c_2, R, q and step size $h \leq \frac{1}{R} \{1/R, 1/q (2c_2/q)^{-1/q}\}$, where $\zeta(q)$ is the Riemann Zeta function.

$$\sum_{n \neq 0} |\hat{f}(\frac{n}{h})| \leq \dots \tag{7}$$

$$\sum_{n \neq 0} |\hat{f}(\dots)|$$

$$S_{\infty}(t) = \frac{h}{\Gamma(\alpha)} \sum_{n \in \mathbb{Z}} e^{(t_0+nh)} e^{-e^{t_0+nh} r} \tag{13}$$

$$\sum_{n \neq 0} \frac{|(t_0+nh)^\alpha|}{\Gamma(\alpha)} < \infty \tag{14}$$

3. Given $\alpha > 0$ and $0 < \beta \leq 1$, for any step size h such that

$$h \leq \frac{2}{3 + (\beta - 1)^{-1} + \beta^{-1}} \tag{15}$$

and any $t_0 \in \mathbb{R}$ we have

$$\frac{|r^\beta - S_{\infty}(t)|}{r^\beta} \leq C, \text{ for all } r > 0, \tag{16}$$

where S_{∞} is given in (13).

$$3,3 \quad 9 \quad 8 \quad (\quad) \quad 4 \quad 0 \tag{16}$$

4. For all $r > 0$,

$$S_F(r) < S_\infty(r) < (1 + \epsilon)r^{-\epsilon}.$$

12. 4. $S_F(r)$, on the whole positive axis, 9.

(16). f (10) r

S_F

5. For any $\epsilon > 0$, $\delta > 0$, and $1 - \epsilon > \delta$, 9.7304 0 0 9.7304 303, ou8309o94 0 TD 0.2518 f4p -33.1058 -2.866 TD 0.0004 0 9.

Let $t_M \leq \dots$ $r \in [0, 1]$ $y_0 = (r^{-1}) \dots$ M $T_M(r)$

$$T_M(t) \leq \frac{r}{(\cdot)} \int_{-\infty}^{t_M} e^{-re^y + y} dy \leq \frac{1}{(\cdot)} \int_{-\infty}^{t_M} e^{-e^y + y} dy \tag{27}$$

$$= \frac{1}{(\cdot)} \int_0^{e^{t_M}} e^{-s} s^{-1} ds = 1 - \frac{(\cdot, e^{t_M})}{(\cdot)}, \tag{28}$$

$$(\cdot, x) = \int_x^{\infty} e^{-s} s^{-1} ds$$

$t_N \geq \dots$ N (-1) t_M (29)

$$T^N(t) \leq \frac{r}{(\cdot)} \int_{t_N}^{\infty} e^{-re^y + y} dy = \frac{1}{(\cdot)} \int_{re^{t_N}}^{\infty} e^{-s} s^{-1} ds,$$

$$r \in [0, 1] \dots T^N(r) \leq \frac{(\cdot, e^{t_N})}{(\cdot)} \tag{30}$$

$$\lim_{x \rightarrow 0} \frac{(\cdot, x)}{(\cdot)} = 1 \quad \lim_{x \rightarrow \infty} \frac{(\cdot, x)}{(\cdot)} = 0,$$

$$1 - \frac{(\cdot, e^{t_*})}{(\cdot)} = \dots \tag{31}$$

$$\frac{(\cdot, e^t)}{(\cdot)} = \dots \tag{32}$$

$t_0, \dots (t_* - t_0)/h, \dots M_*, N_*, S_F, \dots t_*, t^*, h$

7. For all $\dots > 0, \dots > 0$ and $1/e \geq \dots > 0$, the solution t_* of (31) does not depend on \dots and satisfies
$$t_* \geq \frac{(1 + \dots)}{\dots} = \frac{1}{\dots} + \dots (1 + \dots)^{\frac{1}{\dots}} \tag{33}$$

The solution t^* of (32) has a weak dependence on \dots and satisfies

$$t^* \leq \dots^{-1} + \dots^{-1} + \frac{1}{2} \tag{34}$$

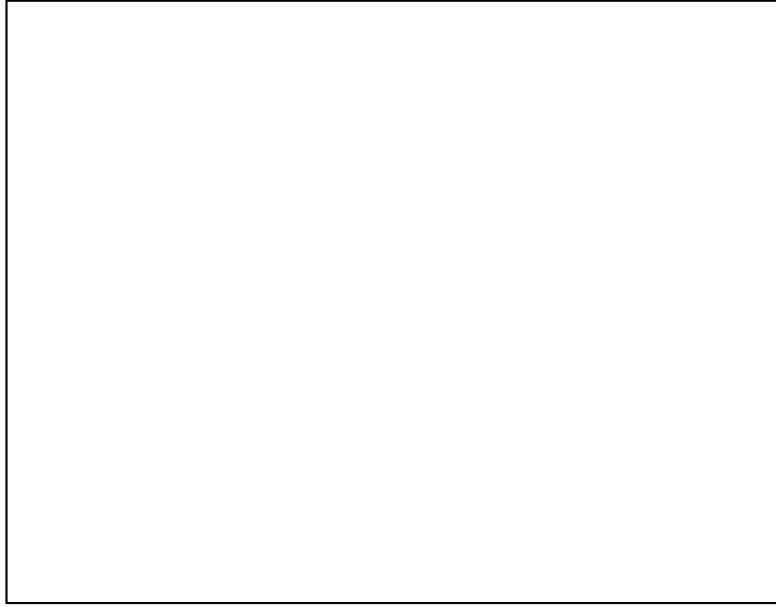
$t_*, t^*, \dots S_F, \dots (33) (34), \dots A, \dots A.2. \dots (9) (10) \dots -\infty \dots +\infty \dots e^{hn} \dots (21) \dots 3 \dots n) A \dots 16, \dots (21)$

8. For any $\epsilon > 0$, and $\delta > 0$, there exist a step size h and a positive integer M such that

$$|e^{-xy} - G_e(x, y)| \leq \epsilon, \quad \text{for } xy \geq \delta, \quad (41)$$

where

$$G_e(x, y) = \frac{hx}{2\sqrt{y}} \sum_{j=0}^M e^{-x^2}$$



be an approximation of the kernel by Gaussians valid for $\frac{1}{2} \leq r \leq 1$. Then, for any bounded, compactly supported function f in D and $x \in D$, we have

$$\left| \int_{B_1} \|\cdot\|^{-r} f(\cdot) d\mu - \int_{B_1} G_F(\|y\|) f(\cdot) d\mu \right| \leq (1 + 2^r)^{d-1} \frac{d-1}{d} \|f\|_\infty.$$

$\tilde{C} \cdot \dots \approx B$

A. $u(z, \cdot) \in C^2(\mathbb{R}^d)$, $z > 0$, $u_{zz} + \Delta u = 0$, $u(0, \cdot) = u_0(\cdot)$, $u(z, \cdot) \rightarrow 0$ as $z \rightarrow \infty$.

$$u(z, \cdot) = \int_{\mathbb{R}^d} \mathcal{P}(z, \cdot - \cdot) u_0(\cdot) d\cdot, \quad z \geq 0, \tag{56}$$

$$\mathcal{P}(z, \cdot) = \frac{2}{d} \frac{z}{(z^2 + \|\cdot\|^2)^{(d+1)/2}}$$

$$S_\infty(z^2 + \|\cdot\|^2) = \frac{zh}{(d+1)/2}$$

$$0 \leq n \leq 2N, \quad a > 0, \quad m = \frac{2N}{a} t_m, \quad [0, 1], \quad h(x) \quad (66)$$

$$\left| h(x) - \sum_{m=1}^M w_m e^{-mx} \right| < \dots \quad (67)$$

$$h \quad (66)$$

- B $(N+1) \times (N+1)$ $0 \leq k \leq N$ $h_k = h_{k+1}$ $h_n = h(a \frac{n}{2N}), 0 \leq n \leq 2N.$
- $(u_0, \dots, u_N),$ $(16, \dots, 22);$ $M+1$ $M \approx \dots$ $M = \mathcal{O}(N^{-1})$
- C m $u(\dots)$

$$x = \frac{1}{2} - \frac{1}{2} \cos \theta, \quad \theta \in [0, \pi] \tag{70}$$

$$\tilde{t}^* = \frac{1}{2} - \frac{1}{2} \cos \theta^* \tag{70}$$

> 1 , ...

$$\frac{(\cdot, x)}{(\cdot)} \leq \frac{e}{(\cdot)} x^{-1} e^{-x} \leq \dots$$

$$\dots \text{ fi } \dots \tag{69}$$

$$x > d(e - 1) \tag{71}$$

$$d = e/(e - 1) \approx 1.582, \dots, 14, \dots, x \dots$$

$$x \geq d$$

A.3. Proof of Theorem 5

2.1.1 $t_0 = 0$ (20) h (15) t_* t^* 7.

$$\tilde{h} = \frac{10}{2^{-1} + 2^1}$$

$$\frac{\tilde{t}^* - \tilde{t}_*}{\tilde{h}}$$

\tilde{t}^*, \tilde{t}_* (33) (34).

$$\tilde{t}^* - \tilde{t}_* = -1 + \frac{1}{-1 + \dots} -1 + \dots -1 + \dots \left(\frac{-1 + 1}{(1 + \dots)^1} \right) + \frac{1}{2}$$

$$\leq -1 + \frac{1}{-1 + \dots} -1 + \dots -1 + \frac{3}{2}$$

15. (23) (27) (28).

15. Let $g(x) = \frac{-(x+1)^{\frac{1}{x}}}{x+1}$ for $x > 0$

33. ... B 0 ... 0 ...

34. ... C ... 121 (7) (2004) 2866–2876. ... B 0 ... 0 ... C ... 121 (14) (2004) 6680–6688.

35. ... 0 ... A ... C ... 20 (2) (1999) 699–718.