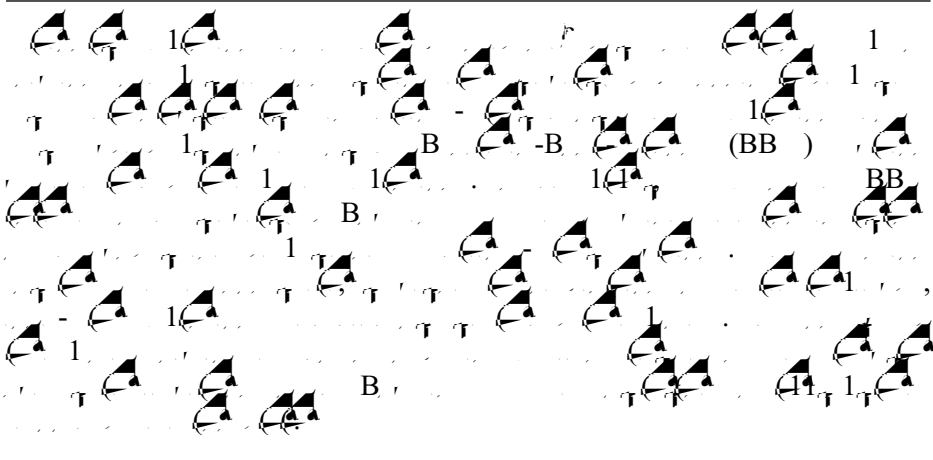
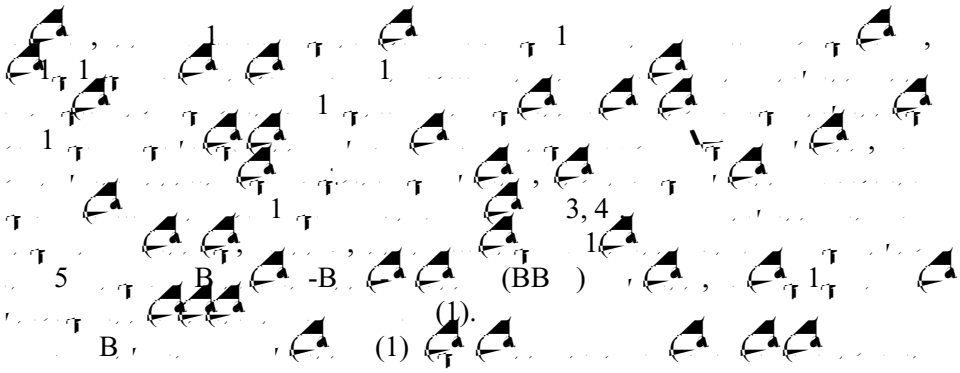



Spin Exchange for a Regulated Bosonic System

By Gennady A. El, Mark A. Hoefer , and Michael Shearer



1. Introduction



(3)  $1_{\sigma} u_{\pm} \dots h_{\pm}$

$$u_{\pm} = h_{\mp} \left(\frac{2}{h} \right)^{k-2} \quad (4)$$

(2) (3)  $1_{\sigma} \dots 1_{\sigma} \dots 1_{\sigma}$ (1)

2. Expansion shock Riemann data



$$h_t \quad ($$

$$h_{\pm} = \frac{1}{16}(r_{\pm} - s_{\pm})^2$$

$$A_{\pm} = \frac{1}{4}(r_{\pm} + s_{\pm})$$

$$H = 1$$

$$1$$

$$(10),$$

$$(14)$$

$$(15).$$

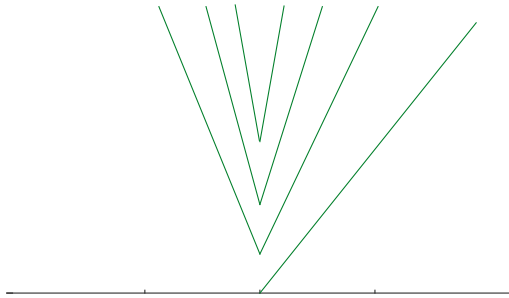
$$(10)$$

3. BBM approximation and the structure of the expansion shock

$$1$$

$$(11)$$

$$1$$



... (17) ...

4. Expansion shock for the Boussinesq equations

... (17) ...

... (13) ...

... (17) ...

... (17) ...

... (17) ...

4.1. Inner solution: first-order approximation

... (17) ...

... (17) ...

$$r^{(1)} = \frac{1}{4\delta} (3r^{(0)} - s^{(0)})r^{(0)} - \frac{1}{6\delta^2} (r^{(0)} - s^{(0)})$$

$$s^{(1)}$$

$$1 \left(\begin{matrix} r^{(0)}, r^{(1)}, r^{(2)}, s^{(0)}, s^{(2)} \\ \vdots \\ 1 \end{matrix} \right) \rightarrow 0, \delta \rightarrow 0, \rightarrow 0. \quad (15)$$

$$\dots \left(\begin{matrix} r^{(1)} \\ \vdots \\ 1 \end{matrix} \right) \dots \left(\begin{matrix} r^{(2)} \\ \vdots \\ 1 \end{matrix} \right) \dots \left(\begin{matrix} s^{(1)} \\ \vdots \\ 1 \end{matrix} \right) \dots \left(\begin{matrix} s^{(2)} \\ \vdots \\ 1 \end{matrix} \right) \dots \quad (20), \dots \quad (21)$$

$$\left(\begin{matrix} r^{(1)} \\ \vdots \\ \dots \end{matrix} \right) \frac{1}{4\delta} \left(\begin{matrix} 3r^{(0)} & s^{(0)} & 3r^{(1)} & s^{(2)} & \dots \end{matrix} \right) \left(\begin{matrix} r^{(1)} & r^{(2)} & \dots \end{matrix} \right) \frac{1}{6\delta^2} \left(\begin{matrix} r^{(1)} & r^{(2)} & s^{(2)} & \dots \end{matrix} \right) \quad (22)$$

$$\left(\begin{matrix} s^{(2)} \\ \vdots \\ \dots \end{matrix} \right) \frac{1}{4\delta} \left(\begin{matrix} r^{(0)} & 3s^{(0)} & \dots \end{matrix} \right) \left(\begin{matrix} s^{(2)} \\ \vdots \\ \dots \end{matrix} \right) \quad (23)$$

$$\frac{1}{6\delta^2} \left(\begin{matrix} r^{(1)} \\ \vdots \\ \dots \end{matrix} \right) \ll k < \delta, \dots \quad (22)$$

$$\left(\frac{r}{\delta} \right) : \frac{1}{\delta}$$

$$K / 0 \quad \frac{2a}{9a^2} \quad \frac{ff'}{f''} \quad K. \quad (30)$$

$$a(\cdot) = \frac{A}{\frac{9}{2}AK - 1} \cdot f(\cdot) \quad B \left(\frac{B}{2K} \right) \quad (31)$$

$$B - 1 \cdot K = \frac{1}{2}. \quad (32)$$

$$a(\cdot) = \frac{A}{\frac{9}{4}A - 1} \cdot f(\cdot) \quad (33)$$

$$r^{(0)}(\cdot) = \frac{s^{(0)}}{3} \pm \frac{\sqrt{A}}{\frac{9}{4}A - 1} \quad (\leftarrow^2). \quad (34)$$

$$s^{(0)}(\cdot) = s^{(0)} \quad (\leftarrow^2) \quad (35)$$

$$r_{\pm} = \frac{s^{(0)}}{3} \pm \sqrt{A} \quad (\leftarrow^2). \quad (36)$$

s_{\pm}

$$\begin{aligned}
 & \gg x = 0, \quad \frac{1}{4}(s^{(0)} - 3r) \quad (19). \\
 & \frac{1}{BB} \quad (18). \\
 & BB \quad 1 \quad 5.
 \end{aligned}$$



$$r^{(c)}(\cdot) \sim 1 - \epsilon \left(\frac{1}{4} - \frac{\epsilon^{(c)}(\cdot)}{1 - \frac{9}{4}} \right) \\ \frac{\epsilon^2}{3} \left(C - \frac{2 - 17 \epsilon^{(c)2}(\cdot) - D \epsilon^{(c)}(\cdot) - E \epsilon^{(c)}(\cdot)}{16 \left(1 - \frac{9}{4}\right)^2} \right).$$

$$s^{(c)}(\cdot) \sim 3 - \frac{3}{4} \epsilon \left(C - \frac{3 \epsilon^{(c)2}(\cdot)}{16 \left(1 - \frac{9}{4}\right)^2} \right)$$



$$r^{(1)} - \frac{1}{4}(3r^{(1)} - s^{(1)})r_X^{(1)} = 0. \tag{51}$$

$$s^{(1)} - \frac{1}{4}(r^{(1)} - 3s^{(1)})s_X^{(1)} = 0$$

$$s^{(1)}(X_1) = 3 \frac{3}{4} - \frac{1}{32} \dots \tag{52}$$

$$r^{(1)}(X_1) = 1 - \left(\frac{1}{4} r_1(X_1) - \frac{1}{96} r_2(X_1) \dots \right)$$

$$r^{(1)}(X_1) = 1 - \left(\frac{1}{4} r_1(X_1) - \frac{1}{96} r_2(X_1) \dots \right) \tag{53}$$

$$r_1 - \frac{3}{4}r_1r_{1.X} = 0. \tag{53}$$

10.95892.701 2.031441.804 . (()20.1()0.2()-330.8()-330.8()-331

From (58), (50), and (59), we have

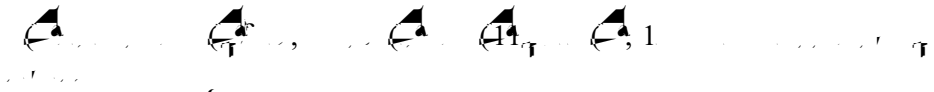
$$r_2(X) \underset{X \rightarrow 0^\pm}{\sim} F_2^\pm \frac{1}{\left(1 - \frac{9}{4}\right)^2} \rightarrow_{\pm} r^{(2)}(X) = \frac{1}{24\left(1 - \frac{9}{4}\right)^2}. \quad (60)$$

For F_2 , $F_2 < 24$, we have

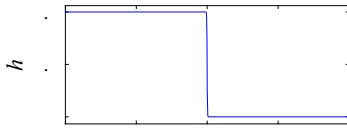
$$r_2(X) = \frac{1 - 3X}{24\left(1 - \frac{9}{4}\right)^2}. \quad (61)$$

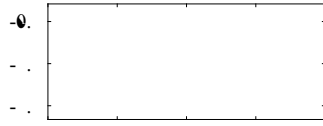
From (61), we have

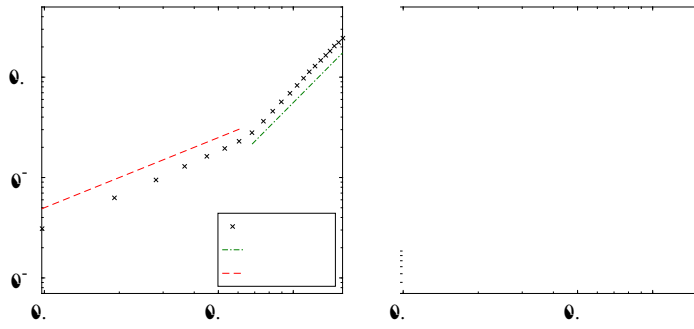
$$r^{(1)}(X) = 1 - \left(\frac{1}{4} - \frac{X - 3X}{1 - \frac{9}{4}} \right) \\ \approx \frac{1}{24} \left(\frac{1}{4} - 1 - 3 \right)$$



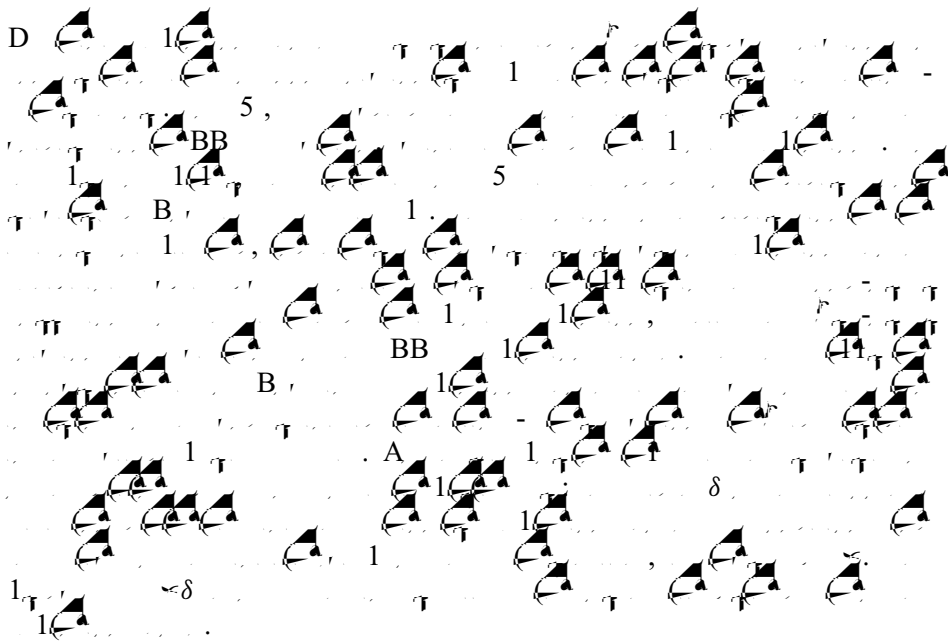
$$r^{(0)}(X, \cdot) = 1 \quad \left\{ \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right.$$







6. Discussion



Acknowledgments

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Appendix

A.1. Let $u(x, t)$ and $h(x, t)$ be the solutions of the initial-boundary value problem (1.1)–(1.4) with $u(x, 0) = u_0(x)$ and $h(x, 0) = h_0(x)$. Then, the solutions $u(x, t)$ and $h(x, t)$ can be written as follows:

$$\begin{aligned} & g_t - (ug)_x - (h_x)_x = 0, \\ & g_t - (u_x)_x - g_x = \frac{1}{3} u_{xxt} = 0. \end{aligned} \tag{A1}$$

B. Let $h(\pm L, t) = h_{\pm}$ and $u(\pm L, t) = u_{\pm}$. Then, the solutions $h(x, t)$ and $u(x, t)$ can be written as follows:

$$\mathcal{F}(ug)_x = ik_n \mathcal{F}ug_n, \quad n = N-2, \dots, N-2-1. \tag{A2}$$

Let $h(x_n, t) = h$ and $g(t) = \frac{1}{2L}(h - h_0)(x_n - L)$. Then, the solutions $h(x, t)$ and $g(t)$ can be written as follows:

$$h(x_n, t) = h + \mathcal{F}^{-1} g(t) = \frac{1}{2L}(h - h_0)(x_n - L). \tag{A3}$$

Let $g_n(t) = \begin{cases} \frac{2L}{N} \sum_{m=N-2}^{N-2-1} x_m g(x_m, t) & n = 0 \\ \frac{g_n(t)}{ik_n} & n = 0 \end{cases}$. Then, the solutions $g_n(t)$ and $u(x, t)$ can be written as follows:

The diagram consists of several rows of elements. The top row features a '1' followed by a series of small 'T' symbols. Below this, there are more '1's and 'T' symbols, some with arrows pointing to them. A '3' is also present. In the middle row, there is a '2' followed by a '3', and 'N' followed by '2¹⁴'. To the right, there is a '45.' and 'L 120'. The bottom row shows '1' and 'T' symbols. A large, faint watermark is visible in the background.