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Three-dimensional inverse scattering for the wave equation with variable speed: near-field formulae using point sources

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Abstract. We consider the inverse scattering problem for the wave equation with variable speed where the region of interest is probed with waves emanating from point sources. We obtain a three-dimensional trace-type formula, which gives the unknown speed in terms of data and the interior wavefield.

1. Introduction

In this paper we consider the inverse scattering problem for the wave equation with [redacted]

Here x and y are points in $\boldsymbol{R}^3$, k is a real scalar, and δ is the three-dimensional delta function. The index of refraction $n(x)$ we assume to be a positive, bounded, real-valued function which is identically one outside some bounded region Ω.

We are interested in particular solutions of (2.1) which we specify with the help of the functions

[redacted]

which satisfy

$$(\nabla^2+k^2)G_0^{\pm}(k,x-y)=\delta(x-y). \qquad (2.2\pm)$$

We now specify solutions G^+ and G^- of (2.1) as solutions of the integral equations

$$G^{\pm}(k,x,y)=G_0^{\pm}(k,x-y)+\int_{\Omega} G_0^{\pm}(k,x-z)k^2V(z)G^{\pm}(k,z,y)\,\mathrm{d}z \qquad (2.3\pm)$$

where $V=1-n^2$.

There are two techniques for showing that $(2.3+)$ and $(2.3-)$ each have unique solutions. One technique [5] shows that for almost every k, (2.3) has a unique solution with $G|V|^{1/2}$ in L^2. Another technique [6], which uses the limiting absorption principle, shows that for every k, (2.3) has a unique solution in a certain weighted Sobolev space. Both these techniques apply in the present case when V is bounded and has compact support.

[redacted]

Proof. The proof, based on the use of Green's formula, is similar to the correspond-[redacted]

$$\int_{\partial\Omega}\left(G_0^-(z-x)\frac{\partial}{\partial\nu}G_0^+(z-y)-G_0^+(z-y)\frac{\partial}{\partial\nu}G_0^-(z-x)\right)\mathrm{d}S_z$$

$$=G_0^-(y-x)-G_0^+(x-y). \tag{2.6}$$

Proof. We set $n(x)$ equal to one in theorem 1.

Corollary 2. Suppose the hypotheses of theorem 1 hold and $G^\pm=G_0^\pm+G_{\mathrm{sc}}^\pm$. Then

$$G_{\mathrm{sc}}^-(y,x)-G_{\mathrm{sc}}^+(x,y)=\int_{\partial\Omega}\left(G_0^-(z-x)\frac{\partial}{\partial\nu}G_{\mathrm{sc}}^+(z,y)-G_{\mathrm{sc}}^+(z,y)\frac{\partial}{\partial\nu}G_0^-(z-x)\right.$$

[redacted]

Proof: We substitute $G^+ = G_0^+ + G_{sc}^+$ into (2.1), simplify, and evaluate at $k = 0$.

Next, we need some time-domain information. Accordingly, we consider the distributional Fourier transforms of G^+ and G^-:

Proof of theorem 2. Following [1] we apply support information to the computation of

[redacted]

[1] Rose J H and Cheney M 1987 Self-consistent equations for variable velocity three-dimensional inverse scattering *Phys. Rev. Lett.* **59** 954–7
[2] Cheney M, Rose J H and DeFacio B 1988 A fundamental integral equation of scattering theory *SIAM*

[redacted]