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Attraction, merger, reflection, and annihilation in magnetic droplet soliton scattering

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The interaction behaviors of solitons are defining characteristics of these nonlinear, coherent structures. Due to recent experimental observations, thin ferromagnetic films offer a promising medium in which to study the scattering properties of two-dimensional magnetic droplet solitons, particle-like, precessing dipoles. Here, a rich set of two-droplet interaction behaviors are classified through micromagnetic simulations. Repulsive and attractive interaction dynamics are generically determined by the relative phase and speeds of the two droplets and can be classified into four types: (1) merger into a breather bound state, (2) counterpropagation trapped along the axis of symmetry, (3) reflection, and (4) violent droplet annihilation into spin wave radiation and a breather. Utilizing a nonlinear method of images, it is demonstrated that these dynamics describe repulsive/attractive scattering of a single droplet off of a magnetic boundary with pinned/free spin boundary conditions, respectively. These results explain the mechanism by which propagating and stationary droplets can be stabilized in a confined ferromagnet.

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Solitary waves or solitons are particle-like wave packets that arise in a wide range of physical contexts from a balance between dispersive spreading and nonlinear focusing. One of the key phenomena that differentiates nonlinear coherent structures such as solitons from their linear counterparts is what happens when such structures interact. Soliton solutions of equations with very special mathematical structure (integrability) have been shown to interact elastically [1] and can be attractive or repulsive [2]. In more general systems, soliton interactions can be significantly more complex, exhibiting fusion, fission, annihilation, or spiraling [3,4]. A relative phase between the solitons plays a dominant role in determining the resulting interaction behaviors. An additional interaction feature, 90 scattering, has been predicted for two-dimensional (2D) magnetic solitons [5,6] and solitons in field theories [7,8]. The recent experimental observation of a magnetic droplet soliton]or linear motion of vortex pairs with net zero topological charge. Perpendicular scattering of two interacting vortex pairs has been theoretically demonstrated [6]. It appears that 90 scattering has a more universal character [8], not requiring a topological charge, and previous numerical studies have indeed shown perpendicular scattering even for approximate nontopological solitons [5]. Loosening topological restrictions and the fact that droplets, due to their precessional nature, possess an extra degree of freedom (phase) opens up many fascinating modes of interaction.

In this Rapid Communication, we classify head-on and angled droplet interactions in terms of the droplets' relative phase and momenta via micromagnetic simulations. Sufficiently in-phase droplets experience an attractive interaction that results in either merger into a new breathing bound state for low speeds, or a scattering event transferring droplet motion to the axis of symmetry. Out-of-phase droplets experience a repelling interaction that results in a scattering event obeying the law of reflection. Via symmetry, these results show that a ferromagnetic boundary with free (pinned) spins attracts (repels) a single droplet. In particular, this provides an explanation for the existence of "edge droplets" theoretically predicted for a spin torque driven, confined ferromagnet with a free spin boundary [13]. Finally, at an intermediate relative phase, the colliding droplets exhibit an "explosion" into spin waves and the spontaneous formation of a single, breathing droplet. This annihilation behavior mimics particle colliders in which high-energy particles are smashed into byproducts.

The model we consider is the Landau-Lifshitz torque equation with perpendicular anisotropy,

$$\frac{\mathbf{m}}{t} = \check{S}\, \mathbf{m} \times [\ {}^{2}\mathbf{m} + (m_z + h_0)\mathbf{z}], \tag{1}$$

where $\lim_{|\mathbf{x}|}$ $\mathbf{m} = \mathbf{z}$. Equat01Tf.44360TD-.00,79Tufi3678([-23.438o3

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dicular anisotropy constant. Here it is assumed that $Q > 1$ or equivalently that the perpendicular anisotropy is sufficiently strong that it overcomes the effective planar anisotropy due to the magnetostatic field. This assumption is not an excessive restriction as ferromagnets with this property such as CoFeB or Co/Ni multilayers are currently in use (cf. [9]). The energy $\mathcal{E} = \frac{1}{2}\int[|\nabla \mathbf{m}|^2 + 1 - m_z^2 + h_0(1 - m_z)]d\mathbf{x}$ is conserved by solutions of (1). The magnetic field induces a positive shift of precession frequency. By entering the rotating frame, we take $h_0 = 0$ without loss of generality.

Droplet solutions of Eq. (1) are parameterized by six distinct quantities: initial position $\mathbf{x}^0$, initial central phase ^{0}, propagation velocity $\mathbf{V}$, and rest precession frequency [14]. A previous droplet interaction study was limited to accurately computed stationary (radially symmetric) droplets [5]. These solutions were artificially deformed to induce propagation with a fixed but not prescribed speed and were accompanied by radiation. Only in-phase, head-on, approximate droplet inter-

The next class of interactions we investigate are propagating droplets with equal frequency $_1 = \quad _2 = \quad$, equal speed $V_1 = V_2 = V$, and velocities reflected, $V_{1,x} = Š\, V_{2,x}$, about the y axis so that **y** represents the axis of symmetry. When the angle of interaction $\quad = \quad$, the collision is head-on. The attractive interaction $| \quad | < \quad _{cr}$ leads to merger and “trapped” scattering along the y axis as in Fig. 1(c). For the symmetric case when $V_{i,y} = 0$, the scattering is 90 . For the repelling interaction, $_{cr} < | \quad | \leqslant \quad$, the droplets reflect at an angle equal to the angle of incidence $\quad / \ 2$, as in Fig. 1(b). Both Figs. 1(b), 1(c) have $\quad = 0.4$, $\quad = 2\ /\ 3$, $V = 0.6$, and successive plotted droplets are $t = 10$ units apart. As $| \quad |$ approaches $_{cr}$, the two droplets collide with one preferentially absorbing the other, transferring a significant portion of their energy into spin waves followed by the spontaneous formation of a breather state as shown in the head-on collision of Figs. 1(d)–1(g) with $\quad = \quad _{cr} = 92$, $\quad = 0.4$, $V = 0.6$, and $t \quad$ (30,40,80,164). The asymmetry in the interaction of Figs. 1(e)–1(g) is due to the choice $0 < \quad < \quad$. A change in the sign of reverses the asymmetry. Figure 1(g) (inset) demonstrates a steep depletion of the excitation amplitude $1\ Š\ m_z$ during the loss of energy to spin waves and an amplitude coalescence associated with the formation of the breather. Annihilation therefore represents the crossover from attractive to repulsive scattering where the incommensurate phases of the colliding droplets cannot be resolved at high kinetic energies, resulting in the explosive release of spin waves accompanied by breather bound state formation.

Previous observations of soliton annihilation in optics were of a very different type [18] where the simultaneous collision of three solitons could result in annihilation of only one of them. Here we see interaction behavior reminiscent of high-energy particles in a collider. The by-products of droplet collision are

[16] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevB.89.180409 for animations of Figs. 1(a)–1(g).

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