

Program in Applied Mathematics  
PROBABILITY AND STATISTICS PRELIMINARY EXAMINATION  
August 23, 2017

Notice: Do four of the following five problems. Place an X on the line opposite the number of the problem that you are **NOT** submitting for grading. Please do not write your name anywhere on this exam. You will be identified only by your student number, given below and on each page submitted for grading. Show all relevant work.

1. \_\_\_\_\_  
2. \_\_\_\_\_  
3. \_\_\_\_\_  
4. \_\_\_\_\_  
5. \_\_\_\_\_  
Total \_\_\_\_\_

Student Number \_\_\_\_\_

1. At the beginning of the semester, an APPM student sorted alphabetically his  $n$  textbooks on a rack. As the semester went by, however, he kept placing each book back on the rack at a random location after consulting it. Let  $p_n$  be the probability that at the end of the semester no textbook ends at its original location (on the rack). Furthermore, let  $q_n$  be the conditional probability that no textbook ends at its original location given that the first textbook on the rack, say book A, does not either.
  - (a) Determine  $p_1$  and  $p_2$ .
  - (b) Explain why  $p_n = \frac{n-1}{n} q_n$ , and  $q_n = \frac{1}{n-1} (p_{n-2} + p_{n-1})$ .
  - (c) Determine a recursion for  $(p_n - p_{n-1})$  and use it to compute  $p_n$  explicitly.

(c) Theorem. If  $(X_n)_{n \geq 0}$  is an i.i.d. sequence of random variables such that  $E(X_i) = 0$  and  $V(X_i) = \sigma^2$ , with  $0 < \sigma < 1$ , then

$\mathbb{P}$

$i=1$