A. Gross-Pitaevskii and the Navier-Stokes equations

II. EXPERIMENT

 $I \qquad \star d$ $N_{\overline{m}} \star d$, W we have $\mathbb{N}_{\overline{w}}$ and \mathbb{N} and \mathbb{N} and \mathbb{N} BEC . In contrast pulses in BECs. In contrast to the theorem in BECs. In contrast to the theorem in BEC experiments described in $[3]$ $[3]$ $[3]$, the experiments and \sim $[3]$, the experiments and \sim in this model with non-rotating condensation \mathbb{R}^d and \mathbb{R}^d are all \mathbb{R}^d . have succeeded in directly imaging dispersive shock waves in the particular geometry of th $y \begin{bmatrix} 1 \ 1 \end{bmatrix}$ in the set of $\begin{bmatrix} 1 \end{bmatrix}$ C \mathcal{A} by 3.5 million Rbs. we \mathbb{R} with trapping \mathbb{R} frequencies of , *^z*-=28.3,5.3- Hz; is the radial $f(x) = \frac{d}{dx}$ *y* $\frac{d}{dx}$ $\frac{d}{dx}$ $\ensuremath{\text{w}}$

z axis through the BEC. For the two the two the two terms of the two

–

td 57.4 205.1 -305.1 -305.1 -305.1 -305.1 -305.1 J *w -339.2 -339.213.5-339.2 -33 4 307. 5 -1.07. -305.17. 4 558-4847. BEC4847. 1 -7. -7. 480.7 -253.7 16. 4 /F4 12.2987 0 444 0 -C9

IV. CLASSICAL AND DISPERSIVE SHOCK WAVES

 $u(x,t)$ w (4.4) (4.4) (4.4)

$$
w(4.4)
$$

$$
\frac{d}{dt} \int_{a}^{b} u(x,t)dx + \frac{1}{2} [u(b,t)^{2} \quad u(a,t)^{2}] = 0, \qquad (4.6)
$$

y *a, b,* $\infty < a < b < \infty$

B. Dispersive shock waves, Korteweg–de Vries equation

A
\n•d
\nK
\nW
\n•d
\n
$$
u_t + \left(\frac{1}{2}u^2\right)_x = \varepsilon^2 u_{xxx},
$$
\n
$$
(4.12)
$$
\n
$$
\varepsilon^2 \ll 1.
$$

I_Pw

$$
u(x,0;\varepsilon) = \begin{cases} 1, & x \neq 0 \\ 0, & x > 0 \end{cases}
$$
 (4.13)

 $\varepsilon^2\to 0.$

4d
\n4d
\nF 8 *d
\nw
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$$
\begin{array}{ccccccccc}\n(4.13) & & & & \varepsilon^2. & \text{O} & & & \star \mathbf{d} & & & \\
\mathbf{w} & \mathbf{w} & & & & \star \mathbf{d} & & \varepsilon & \mathbf{f} & & 1585-499.7 & .4 & \star \mathbf{d} & & J/F41 & (4.12221 & 0.01 & 0.3330 & 94.13 & /1.2315 & 610.5062 & & & \\
\end{array}
$$

$$
L = 2K(m)\sqrt{\frac{6}{r_3 - r_1}},
$$

where $K(m)$ is the first kind of the firs $[25]$ $[25]$ $[25]$. N_c L is obtained the periodicity of the periodicity of the periodicity of the dn p $f(x) = \frac{E}{(4.14)}$ $f(x) = \frac{E}{(4.14)}$ $f(x) = \frac{E}{(4.14)}$, $\frac{E}{(4.14)}$, $\frac{E}{(4.14)}$ = 2*K*(*m*) w [] + d $m \longrightarrow M$ elliptic function. See Fig. [9](#page-9-0) for ϕ *m*. W $\leq \leq \leq \leq \leq \leq \leq \cdots$ d $(y; 1) =$ (y), y w . \mathbb{R} idea behind which which which which \mathbb{R} \overline{u} **b** \overline{u} of Eq. [4.12](#page-8-1) \overline{z} of \overline{z} is assumed to \overline{u} $h = \frac{1}{1}$ in Eq. [4.12](#page-8-1)), $= (x + 1)^2$ $Vt)/ε$
+**d** $\sqrt{\varepsilon}$ and a fast variable. The numerical computation $-\varepsilon$ tations picture in Fig. [8](#page-8-0) subset that modulations of the third \mathbb{R} suggests that modulations of the this subset of this subset of this subset of the this subset of this subset of this subset of this subset of this su \mathbf{v} *x* and *t*, i.e., $\{r_i = r_i(x,t)\}, i = 1,2,3.$

$$
\overline{\phi}(x,t) = \frac{1}{L} \int_0^L \phi(\cdot, x, t) \, dt = r_1(x,t) + r_2(x,t) \quad r_3(x,t)
$$
\n
$$
+ 2[r_3(x,t) \quad r_1(x,t)] \frac{E[m(x,t)]}{K[m(x,t)]},\tag{4.15}
$$

is the complete element of the second o

w $E(m)$

4d [25].
\n
$$
\begin{aligned}\n & \mathbf{K}\mathbf{d} = \mathbf{W} \quad \mathbf{V} \quad \mathbf{V} \\
 & \mathbf{K}\mathbf{V} = \mathbf{V} \quad \mathbf{V} \quad \mathbf{V} \\
 & \mathbf{V} = \mathbf{V} \quad \mathbf{V} \quad \mathbf{V} \\
 & \mathbf{V} = \mathbf{V} \quad \mathbf{V} \quad \mathbf{V} \\
 & \mathbf{V} = \mathbf{V} \quad \mathbf{V} \quad
$$

*r*1*x*,0-  0, *r*2*x*,0- = 0, *x* 0 1, *^x* 0, *^r*3*x*,0-  1, 4.18shown in Fig. [10](#page-10-0) regularizes the IVP [4.12](#page-8-1) and [4.13](#page-8-2) because of the following properties: *¯x*,0- = *ux*,0; characterization-, *ri x x*,0- 0 nondecreasing-, *x*-R *ri x*,0- min *x*-R *ri*+1*x*,0separability-

$$
\sum_{x \in \mathbb{R}} r_i(x,0) < \sum_{x \in \mathbb{R}} r_{i+1}(x,0) \quad (y). \tag{4.19}
$$

 Γ and initial data for verifying that the initial data for \mathbf{y} (4.13) (4.13) (4.13) is equivalent to the initial data for the initial dat $\overline{\phi}$; the same assumption is made in the same assumption is ma \mathfrak{sd} y- \mathfrak{sd} [[23](#page-23-4)]. \mathfrak{sd} s_p and r_i ensure that a global r_i ensure that a global ϵ $\left(4.17\right)$ $[18,26]$ $[18,26]$ $[18,26]$ $[18,26]$. y (4.17) w \star **d** (4.18) (4.18) (4.18) $\mathbf{4.18}$ rarefaction solution in the form of a self-similar simple wave w $r_1 = 0, r_3 = 1, \quad \text{if } r_2(x,t) = r_2(\xi), \xi = x/t.$

$$
(4.17)
$$

$$
(v_2 \t \xi) r'_2 = 0,
$$

 \cdots

4givn a DSWse Fig712and Eq7823.DISPERV AND CLASI SHOCK WAVES INºPHYSCAL REVIW A74, 0236232006023623-11

$$
v_2 = \xi
$$

FIG. 117 2satifynghe jump condit Colr onlieDspatvdhen dispersiveoldregularizatonsoftheconservationlaw874. Thedispatvec correspond to a traveling discontuity with sped 879. The dispersive case is a rarefaction-wave 1v2 solution to theWiam equations87

= 1. This rarefaction wave modulates thepriodc solution87

rFIG.107uColronlieItadregularizatonfrtheKdV 2r3ispersiveRmanproblem. Thedaslinrepresnthial

$$
\frac{1}{3}[1+r_2(\xi)] \frac{2}{3}r_2(\xi) \frac{[1-r_2(\xi)]K[r_2(\xi)]}{E[r_2(\xi)] [1-r_2(\xi)]K[r_2(\xi)]} = \xi,
$$
\n(4.20)

$$
r_{2}(\xi), \text{ w} \qquad \text{ s.d. } y \qquad \text{ s.d. } s \qquad \text{ s.d. } w ,
$$
\n
$$
\xi \quad (\text{ F . 11}).
$$
\n
$$
v_{2}^{+}, \text{ w} \qquad \text{ s.d. } s \qquad \text{ s.d. } w \qquad \text{ s.d. } v_{2} \qquad \text{ s.d. } v
$$
\n
$$
(4.17) \text{ A} \qquad \text{ s.d. } v
$$

4.17-. Ahead of the moving fronts, the *ri* are constant. Since

$$
\frac{dr_2}{dt} = 0 \quad \text{w} \qquad \frac{dx}{dt} = v_2,
$$

from Eqs. [4.17b](#page-9-2)-, the speeds are given by the limits

$$
v_2^+ = \n \quad v_2(0, r_2, 1) = \frac{2}{3},
$$
\n(4.21)

$$
v_2 = \sum_{r_2 \to 0^+} v_2(0, r_2, 1) = 1.
$$
 (4.22)

 $H = \int_{0}^{1} (4.12) w$ $H = \int_{0}^{1} (4.12) w$ $H = \int_{0}^{1} (4.12) w$ d (4.13) (4.13) (4.13) the $\varepsilon^2 \ll 1$, the leading asymptotic equations of \mathbf{y} (4.14) \star d (4.17) w \star d (4.18) (4.18) (4.18) , y $(\varepsilon^2 \ll 1, t \ll 1/\varepsilon)$ D

3foru.The solid lines represnt the intal dat for theRimanvriantsr1,r2,andr3tharegularizethnaldforu.This intal dat for the Rieman invariants satiÝe the threproperties of characterizaton, nodecreasing, and separabilty87

$$
u(x,t;\varepsilon) \sim r_2(x/t) \quad 1 + 2 \cdot d \frac{2\left(\frac{x}{\varepsilon\sqrt{6}}\right) f(x/t)}{(\varepsilon\sqrt{6})} ; m = r_2(x/t) \bigg),
$$

$$
V(x/t) = \frac{1}{t}
$$

which a constant connect stop a train of second $\frac{2}{\pi}$ solitons eventtually leading to small, the tail of tails at the tail of tails \mathbf{I} at the tail of tails \mathbf{I} at the tail. $\bf w$ and a dispersive shock of $\bf w$ wave associated with Kd in the context of M in the context of A D can arise in the dispersive regularization of a con- \mathbf{w} is a constraint in the dissiw. $y * d$ weak limit w
 ***d *d** \bullet scillations is required in the dispersive case. This method is \bullet gives useful results such as the asymptotic modulated oscil-such as the asymptotic modulated oscillatory profile, the wavelength of oscillation, the leading amplitude of oscillation, the leading amplitude amon plitude speeds of a speed shock. On a large shock \mathbf{d} scale, once the limiting process has been accomplished, the limiting process has been accomplished, the limiting \mathbf{S} $D \longrightarrow \mathbb{R}$ $\begin{array}{ccc} \sim & , & \ldots, \\ \mathbf{y} & \mathbf{y} & , \mathbf{y} & , \end{array}$ $\frac{d}{d}$
 $\frac{d}{d}$ O and $(L \rightarrow kl L \quad [31]$ $(L \rightarrow kl L \quad [31]$ $(L \rightarrow kl L \quad [31]$ used the inverse scattering transform to take the limit $\varepsilon \to 0$ $i = \frac{Kd}{v}$ ([4.12](#page-8-1))
y w+d \bar{u} $f(x) = \frac{1}{2} \int_0^x \frac{dx}{y} dx$ \overline{u} we we $\int_{\varepsilon \to 0}^{\infty} \int_{-\infty}^{\infty}$ ∞ $u(x,t;\varepsilon)f(x)dx = \int_{-\infty}^{\infty}$ $\overline{u}(x,t)f(x)dx$ for all smooth, compactly defined functions *fx*-. y limiting procedure is required because the solution develops of \ast of \ast and infinite number of \overline{u} and \overline{u} and \overline{u} and \overline{u} and \overline{u} \bar{u} ed y y using the Whiteham averaging the Whiteham averaging the Whiteham averaging the Whiteham averaging the W method thus giving the Whitham method a stronger mathematical footing. \mathbf{w} , we will use the rest of this paper, we will use the rest of this paper, we will use the rest of this paper. dissipative-dispersive analogy with the Burgers' and KdV \mathbf{e} is motivate our discussion of the more complication of the mor $\begin{array}{ccc} \bullet & \bullet & \bullet \end{array}$ \rm{BEC} and gas dynamics.

C. Dissipative regularization of the Euler equations

As mentioned in the Introduction, the compressible equations of gas dynamics without dissipation are the same as the local conservation equations for a BEC [1.5](#page-1-1) with =0. Let us consider the Riemann problem for the dissipative regularization of the Euler equations in one dimension with step initial data

 $\rho_t + (\rho u)_x = 0,$ $(\rho u)_t + (\rho u^2 + \frac{1}{2})$ $\left(\frac{1}{2}\rho^2\right)_x$ $= 0,$ $\rho(x,0) = \begin{cases} 1 & x \neq 0 \\ 0 & x = 0 \end{cases}$ $\rho_0, \quad x < 0$

1, $x > 0$, $u(x, 0) = \begin{cases} u_0, & x < 0 \\ 0, & x > 0 \end{cases}$. (4.25)

$$
\rho_0 \quad \text{ with } u_0. \text{ N} \quad , \quad \text{w} \qquad \qquad \text{w} \qquad \text{w} \qquad \text{-}
$$

$$
\widetilde{\rho} = \rho_r \rho, \quad \widetilde{u} = \sqrt{\rho_r} u + u_r,
$$

$$
t = \rho_r \tilde{t}, \quad x = \sqrt{\rho_r} (\tilde{x} \quad u_r \tilde{t}), \quad \rho_r \neq 0,
$$

and
(4.25) (4.26)

$$
\tilde{\rho}(\tilde{x},0) = \begin{cases}\n\rho_l = \rho_r \rho_0, & \tilde{x} < 0 \\
\rho_r, & \tilde{x} > 0\n\end{cases}
$$
\n
$$
\tilde{u}(\tilde{x},0) = \begin{cases}\nu_l = \sqrt{\rho_r} u_0 + u_r, & \tilde{x} < 0 \\
u_r, & \tilde{x} > 0\n\end{cases}
$$
\n
$$
\text{w} \qquad (4.25) \qquad \text{w} \qquad \text{w}
$$
\n
$$
\vdots \qquad \qquad \downarrow \downarrow \qquad \qquad \text{w} \qquad \text{w}
$$
\n
$$
\vdots \qquad \qquad \downarrow \downarrow \qquad \qquad \text{w} \qquad \text{w}
$$
\n
$$
\vdots \qquad \qquad \downarrow \downarrow \qquad \qquad \text{w} \qquad \text{w}
$$

$$
V\phi'A + \frac{1}{2}A'' \frac{1}{2}\phi'^{2}A \quad A^{3} = 0. \tag{4.33}
$$

I $\star d$ $\phi', w \quad \star d$

 $\phi' = V \frac{2c_1}{A^2},$

where c_1 is a constant of integration. Inserting the integration of integration. In this result in

4d E . (4.33) -d y

$$
A'' + V^2 A - \frac{4c_1^2}{A^3} - 2A^3 = 0.
$$

 I

$$
A'^2 + V^2 A^2 + \frac{4c_1^2}{A^2} \quad A^4 + c_2 = 0,\tag{4.34}
$$

where c_2 is a second constant of integration.

$$
ρ2 = 4(ρ3 V2ρ2 c2ρ 4c12) = 4(ρ λ1)(ρ λ2)(ρ λ
$$

$$
\left(\rho u^2 + \rho^2 + \varepsilon^2 \frac{\rho_x^2}{4\rho}\right)_t + \left(\rho u^3 + 2\rho^2 u + \varepsilon^2 \right)^\rho
$$

 $\begin{array}{ccc} \star & \star & \star & \star & \star \end{array}$ $F \cdot 17.E$

$$
B = \sqrt{\rho_0}, \quad \mu = \frac{2 - \sqrt{\rho_0}}{\sqrt{\rho_0}}.
$$
 (4.53)

$$
W = 2\sqrt{\rho_0} \quad 2, \text{ w}
$$

 $\mathsf{sd}(4.51)$ $\mathsf{sd}(4.51)$ $\mathsf{sd}(4.51)$ y w (4.53) (4.53) (4.53) **is** (4.54) (4.54) (4.54)

$$
v_g = \sqrt{\rho_0},
$$
\nE. (4.49), D
\n* d D W * d
\ny W ,
\nKd
\n
$$
\begin{array}{cccc}\n & \text{if } & \text{if } & \text{if } \\
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 & \text{if } & \text{if } & \text{if } & \text{if } \\
\end{array}
$$

I F . [19](#page-18-0)

 $\overline{)}$

$$
u = \varepsilon \left[\quad \Psi(x,t) \right]_x = W \quad \frac{\mu B \beta^2}{B^2} \quad \frac{2}{\left(\quad \right) + \mu^2},
$$

$$
= B \beta [x \quad (B\mu \quad W)t \quad x_0]/\varepsilon, \quad (4.50)
$$

W
(*B*, *W*, μ , β , x_0)
$$
\begin{array}{c} \n\text{if} \quad 18 \text{,} \quad \text{if} \quad \mu^2 + \beta^2 = 1, \\ \n\text{if} \quad 18 \text{,} \quad \text{if} \quad y \n\end{array}
$$

$$
v_g = B\mu \quad W. \tag{4.51}
$$

I^t \mathbf{A} and \mathbf{y} and \mathbf{y} occur when $x=(B\mu)^T$ $\frac{1}{w}$, $\frac{W}{t+x_0}$ $t + x_0$ giving the maximum values of $t + x_0$

$$
\rho = |\Psi|^2 = B^2, \quad \rho = |\Psi|^2 = (B\mu)^2,
$$

$$
u = \varepsilon (\Psi)_x = W,
$$

$$
u = \varepsilon (\Psi)_x = B\left(\mu \frac{1}{\mu}\right) W. \quad (4.52)
$$

 $\mathcal{B}, \mu, \text{ and } W, \text{ w}$ and minimum solution in Eqs. (4.52) (4.52) (4.52) $\ddot{\mathbf{r}}$ maximums of the trailing dip in the trailing dip in the trailing dip in the DSW $\ddot{\mathbf{r}}$

$$
\xi_v = 2\sqrt{\rho_0} \quad 2 \quad 2\left[1 - \frac{(\sqrt{\rho_0} - 1)E(m_v)}{(\sqrt{\rho_0} - 2)K(m_v)}\right]^{-1},
$$
\n
$$
m_v = (\sqrt{\rho_0} - 1)^{-2}. \qquad (4.55)
$$
\n
$$
\text{E} \quad . \text{ (4.45)}
$$
\n
$$
(4.47).
$$
\n
$$
\text{E} \quad . \text{ (4.47)}
$$
\n
$$
(4.47).
$$
\n
$$
\text{E} \quad . \text{ (4.37)}
$$
\n
$$
(4.40) \cdot \text{d} \quad \text{d}
$$
\n
$$
\text{W} \quad \text{d} \quad \text{d} \quad (4.40) \cdot \text{d} \quad \text{d} \quad
$$

E. Theoretical explanation of experiments

I	\n $\star d$ \n $\star d$ \
---	---

$$
\lambda_1(x/t) = \frac{1}{16} [4\sqrt{\rho_0} \quad 6 \quad r_3(x/t)]^2,
$$

$$
\lambda_2(x/t) = \frac{1}{16} [4\sqrt{\rho_0} + 2 \quad r_3(x/t)]^2,
$$

$$
\lambda_3(x/t) = \frac{1}{16} [4\sqrt{\rho_0} \quad 2 + r_3(x/t)]^2, \tag{4.58}
$$

w r_3 ([4.48](#page-16-2)) . F $1 < \rho_0$ $<$ 4, $(x/t) \equiv 1$. ρ ρ_0 4, $\ddot{ }$ $E. (4.56).$ $E. (4.56).$ $E. (4.56).$ H we find that $y \star d$ we find that $\star d$ dissipative shock waves is the method of regularization. In the dispersive case, we use in particularization \mathcal{U} and \mathcal{U} parameters \mathcal{U} are guidarization. In particular, \mathcal{U} $\mathsf w$, $\mathsf w$, $\mathsf w$ initial data, y for all times in the dissipative case, \mathbf{d} entropy conditions are employed. Using a dispersive regularization determined the behavior of a fundamental the behavior of a fundamental the behavior of a fundamental t D in Bose-Einstein condensation conde $D \Box$ speeds and oscillatory behavior are different.

 $\Psi_{1D}(\, x,t=0) = \Psi_{1D}(x,t=0)$ (4.59)

 $C \longrightarrow \mathbb{R}$ \mathfrak{so} y \mathfrak{so} \mathfrak{so} $\mathfrak{sp} \vert \Psi \vert^2$ \mathfrak{so} y VII 3D A velocid $[\Psi_{3D}(r,0,t)]_r$. I 2D, $[\Psi_{2D}(r,t)]_r$ w 1D, $y \left[\Psi_{1D}(x,t) \right]_x$, 3D • $2D$ results are found to be barely distinguishable with less with $y \star d$ 1% difference between them in density. The difference of \mathbf{v} . between them cannot be seen in Fig. [23.](#page-20-0) Also, in Figs. [24](#page-20-1) and 27 , we are $3D$ and $1D$ results are $3D$ and $1D$ $\begin{array}{ccc} 3D & \star \star 1D \\ y & \star \star 1D \end{array}$ in Sec. $I = \frac{1}{2}$, $\frac{1}{2}$, $\begin{bmatrix} 3D \\ w \end{bmatrix}$. V_{it} [E (3.1) (3.1) (3.1)]. \blacksquare . $\tilde{t} = \delta t \quad (5 \quad ,$, $\mathbf{A} \times \mathbf{N}$, \mathbf{W} , red by y GP with w $E = 23$ $E = 23$ is a comparison between the set of \mathbb{R}^n 3D $\frac{F}{(\alpha_r=0.71)^{4d}}$ with $\frac{23}{d}$ and $w = (\alpha_r = 0)$ sion potential. The trailing edge of the DSW propagates to the D $w \rightarrow d$ when the center when the center when the center w $w \hspace{2.2cm}$ with the expansion potential, $\frac{1}{2}$ \mathbf{A} the leading and trailing edges of the DSW propagate \mathbf{D} radially outward. Otherwise, the two simulations are very similar.

tions of the Euler equations of the Euler equations of gas dynamics and a BEC are \mathbb{R} and a BEC ϵ d. The dissipative regularization of the Euler equation of the Euler equality regularization of the Euler $\mathbf w$ and $\mathbf w$ by the finite-volume package $\mathbf w$ - $[20]$ $[20]$ $[20]$ w

 ρ_t + (

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