A. Gross-Pitaevskii and the Navier-Stokes equations

w , ﬁ
E , y
w ﬁ ﬁ ﬁ y
(1.4). I w w w -
ﬁ w
ﬁ . O w ﬁ w
$\rho = 0$ ﬁy . -
(1.5) -
.

II. EXPERIMENT

I ﬁ ﬁ
w ﬁ, w ﬁ w -
BEC . I
ﬁ ﬁ . [3], y ﬁ
w ﬁ w ﬁ .
ﬁ ﬁ ﬁ y ﬁ w
y , ﬁ y -
y -
ﬁ .
C ﬁ y 3.5
w ﬁ y w
$(\ _{\perp}, \ _{z}) = 2\pi(8.3, 5.3)$ H ; $_{\perp}$ ﬁ
y ﬁ $_{z}$ y. A ﬁ
w
z BEC. F w
y ﬁ 57.4 205.1 -305.1w -305.1 -305.1 -305.1 -305.1 J *w -339.2 -339.213.5-339.2 -33
4 307. 5 -1.07. -305.17. 4 558-4847. BEC4847. 1 -7. -7. 480.7 -253.7 16. 4 /F4 12.2987 0 444 0 -C9
(–

BEC. w

y

. A

y , y

\-

. F w ,

w

. B - - y -

w y BEC w

. I y

w

y F . 3 5 w

w y w (D) w

y

w D .

IV. CLASSICAL AND DISPERSIVE SHOCK WAVES

I , w

w . -

-

. -

y y, w BEC

$u(x,t)$ w (4.4)

$$\frac{d}{dt}\int_a^b u(x,t)dx + \frac{1}{2}[u(b,t)^2 \quad u(a,t)^2] = 0, \qquad (4.6)$$

y $a, b,$ $\infty < a < b < \infty$

B. Dispersive shock waves, Korteweg–de Vries equation

A D (. ., [21]), w

K w (K)

$$u_t + \left(\frac{1}{2}u^2\right)_x = \quad \varepsilon^2 u_{xxx}, \qquad (4.12)$$

$\varepsilon^2 \ll 1.$

L

$$u(x,0;\varepsilon) = \begin{cases} 1, & x \quad 0 \\ 0, & x > 0 \end{cases} \qquad (4.13)$$

$\varepsilon^2 \to 0.$

w (4.4) w

.

F 8 w E . (4.12)

w y

(4.13) ε^2. O

w w ε 15 85-499.7 .4 J/F4 1 (4.12 22 1 0 0 1 0.333 0 94.13 /1.2315 610.5062

$$L = 2K(m)\sqrt{\frac{6}{r_3 \quad r_1}},$$

w $K(m)$ ◆d
[25]. N L ◆d ◆d y ◆d
(4.14), . ., $\left[\sqrt{\frac{r_3 \quad r_1}{6}}\right] = 2K(m)$ w [ ] ◆d
. m ◆d
. F . 9 ϕ
m. w ◆d $(y;0)=1$ ◆d
◆d $(y;1)=$ (y), y w .
◆d ◆d y
,,
. y ◆d
w $\bar{u}$ E . (4.12) $\varepsilon \to 0$. ε ◆d
1 E . (4.12), $=(x$
$Vt)/\varepsilon$ ◆d ◆d . -
◆d F . 8 ◆d
◆d w
x ◆d t, . ., $\{r_i = r_i(x,t)\}$, $i = 1,2,3$. ϕ

$$\bar{\phi}(x,t) = \frac{1}{L}\int_0^L \phi(\ ,x,t)d \quad = r_1(x,t) + r_2(x,t) \quad r_3(x,t)$$

$$+ 2[r_3(x,t) \quad r_1(x,t)]\frac{E[m(x,t)]}{K[m(x,t)]}, \qquad (4.15)$$

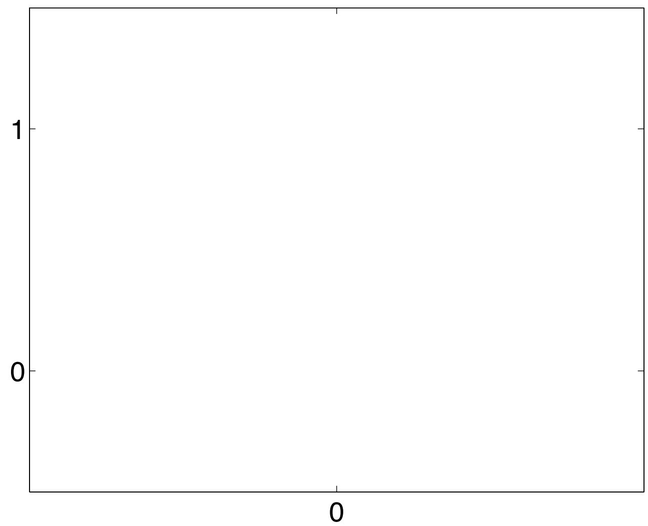
w $E(m)$ ◆d
◆d [25].
w ◆d w
K◆d [22]

$$u_t + \left(\frac{1}{2}u^2 + \varepsilon^2 u_{xx}\right)_x = 0,$$

$$\left(\frac{1}{2}u^2\right)_t + \left(\frac{1}{3}u^3 + \varepsilon^2 u u_{xx} \quad \frac{1}{2}\varepsilon^2 u_x^2\right)_x = 0,$$

$$\left(\frac{1}{3}u^3 \quad \varepsilon^2 u_x^2\right)_t + \left(\frac{1}{4}u^4 \quad 2\varepsilon^4 u_x u_{xxx} + \varepsilon^4 u_{xx}^2 + \varepsilon^2 u^2 u_{xx}\right.$$

$$\left. 2\varepsilon^2 u u_x^2\right)_x = 0. \qquad (4.16)4.16$$



$$r_1(x,0)\equiv 0,\quad r_2(x,0)=\begin{cases}0, & x\quad 0\\ 1, & x>0,\end{cases}\qquad r_3(x,0)\equiv 1, \tag{4.18}$$

w F . 10 I (4.12) (4.13) - w :

$$\overline{\phi}(x,0)=u(x,0;\varepsilon)\quad (\qquad\qquad),$$

$$\frac{\partial r_i}{\partial x}(x,0)\quad 0\quad (\qquad\qquad),$$

$$\underset{x\in\mathbb{R}}{} r_i(x,0) < \underset{x\in\mathbb{R}}{} r_{i+1}(x,0)\quad (\qquad\qquad y). \tag{4.19}$$

C y (4.13) $\overline{\phi}$; y- [23]. y r_i , (-) (4.17) [18,26]. y (4.17) w (4.18) - w w $r_1\equiv 0$, $r_3\equiv 1$, $r_2(x,t)=r_2(\xi)$, $\xi=x/t$. (4.17)

$$(v_2\quad \xi)r_2'=0,$$

w w $v_2=\xi$

$$\frac{1}{3}[1+r_2(\xi)]\quad \frac{2}{3}r_2(\xi)\frac{[1\quad r_2(\xi)]K[r_2(\xi)]}{E[r_2(\xi)]\quad [1\quad r_2(\xi)]K[r_2(\xi)]}=\xi, \tag{4.20}$$

. w , $r_2(\xi)$, w y - ξ (F . 11). w w v_2 v_2^+, w (4.17). A , r_i .

$$\frac{dr_2}{dt}=0\quad w\quad \frac{dx}{dt}=v_2,$$

E . (4.17), y

$$v_2^+=\underset{r_2\to 1}{}\ v_2(0,r_2,1)=\frac{2}{3}, \tag{4.21}$$

$$v_2=\underset{r_2\to 0^+}{}\ v_2(0,r_2,1)=\quad 1. \tag{4.22}$$

, E . (4.12) w (4.13) $\varepsilon^2\ll 1$, y (4.14) (4.17) w (4.18), y $(\varepsilon^2\ll 1, t\ll 1/\varepsilon)$ D

$$u(x,t;\varepsilon)\sim r_2(x/t)\quad 1+2\ {}^2\left(\frac{x\quad V(x/t)t}{\varepsilon\sqrt{6}};m=r_2(x/t)\right),$$

$V(x/t) = \frac{1}{}$

w 2 -
y ,
w w y
w w K y.
A D -
w -
w. y -
weak limit w
.
y -
y , w , -
, . O
, ,
D , . ., -
y . H w ,
.
O , L L [31]
$\varepsilon \to 0$
K (4.12) .
y w $\bar{u}$ w

$$\varepsilon\to 0 \int_{\infty}^{\infty} u(x,t;\varepsilon) f(x) dx = \int_{\infty}^{\infty} \bar{u}(x,t) f(x) dx$$

, y $f(x)$. y
. L L
w , , w $\bar{u}$
y y
-
.
, w w
y w B ' K
-
-
BEC y .

C. Dissipative regularization of the Euler equations

A I , -
y w
BEC (1.5) w $\varepsilon = 0$. L
-
E w

$$\rho_t + (\rho u)_x = 0,$$

$$(\rho u)_t + \left(\rho u^2 + \frac{1}{2}\rho^2\right)_x = 0,$$

$$\rho(x,0) = \begin{cases} \rho_0, & x<0 \\ 1, & x>0 \end{cases}, \quad u(x,0) = \begin{cases} u_0, & x<0 \\ 0, & x>0 \end{cases}. \tag{4.25}$$

- w w -
ρ_0 u_0. N , w

$$\tilde{\rho} = \rho_r \rho, \quad \tilde{u} = \sqrt{\rho_r}\, u + u_r,$$

$$t = \rho_r \tilde{t}, \quad x = \sqrt{\rho_r}(\tilde{x} \quad u_r \tilde{t}), \quad \rho_r \neq 0, \tag{4.26}$$

w (4.25)

$$\tilde{\rho}(\tilde{x},0) = \begin{cases} \rho_l = \rho_r \rho_0, & \tilde{x}<0 \\ \rho_r, & \tilde{x}>0 \end{cases},$$

$$\tilde{u}(\tilde{x},0) = \begin{cases} u_l = \sqrt{\rho_r}\, u_0 + u_r, & \tilde{x}<0 \\ u_r, & \tilde{x}>0 \end{cases}.$$

w (4.25) , ,
w w
: w [-
E . (4.10) $n=2$]. w y

$$V\phi' A + \frac{1}{2}A'' \quad \frac{1}{2}\phi'^2 A \quad A^3 = 0. \tag{4.33}$$

I ϕ', w

$$\phi' = V \quad \frac{2c_1}{A^2},$$

w c_1 . I
E . (4.33) y

$$A'' + V^2 A \quad \frac{4c_1^2}{A^3} \quad 2A^3 = 0.$$

I

$$A'^2 + V^2 A^2 + \frac{4c_1^2}{A^2} \quad A^4 + c_2 = 0, \tag{4.34}$$

w c_2 .
- , $\rho = A^2$; E . (4.34)

$$\rho'^2 = 4(\rho^3 \quad V^2\rho^2 \quad c_2\rho \quad 4c_1^2) = 4(\rho \quad \lambda_1)(\rho \quad \lambda_2)(\rho \quad \lambda$$

$$\left(\rho u^2 + \rho^2 + \varepsilon^2 \frac{\rho_x^2}{4\rho}\right)_t + \left(\rho u^3 + 2\rho^2 u + \varepsilon^2 \rho\right.$$

$$u = \varepsilon[\quad \Psi(x,t)]_x = \quad W \quad \frac{\mu B\beta^2 \quad {}^2(\ )}{B^2 \quad {}^2(\ ) + \mu^2},$$

$$= B\beta[x \quad (B\mu \quad W)t \quad x_0]/\varepsilon, \qquad (4.50)$$

w (B, W, μ, β, x_0) $\mu^2+\beta^2=1$ (F . 18). y

$$v_g = B\mu \quad W. \qquad (4.51)$$

I ' y y w $x=(B\mu$ $W)t+x_0$

$$\rho \quad = \quad |\Psi|^2 = B^2, \quad \rho \quad = \quad |\Psi|^2 = (B\mu)^2,$$

$$u \quad = \quad \varepsilon(\quad \Psi)_x = \quad W,$$

$$u \quad = \quad \varepsilon(\quad \Psi)_x = B\left(\mu \quad \frac{1}{\mu}\right) \quad W. \qquad (4.52)$$

B, μ, W, w y E . (4.52) D

F . 17. E , w

$$B = \sqrt{\rho_0}, \quad \mu = \frac{2 \quad \sqrt{\rho_0}}{\sqrt{\rho_0}}. \qquad (4.53)$$

, $W = 2\sqrt{\rho_0} \quad 2$, w

$$W = 2 \quad 2\sqrt{\rho_0}. \qquad (4.54)$$

(4.51) y w - (4.53) (4.54)

$$v_g = \sqrt{\rho_0},$$

E . (4.49), D - . D w y w , , . . - , K , . [7], D y .

I F . 19

$$\xi_v = 2\sqrt{\rho_0} \quad 2 \quad 2\left[1 \quad \frac{(\sqrt{\rho_0} \quad 1)E(m_v)}{(\sqrt{\rho_0} \quad 2)K(m_v)}\right]^{\ 1},$$

$$m_v = (\sqrt{\rho_0} \quad 1)^{\ 2}. \tag{4.55}$$

ρ_0 4,

E . (4.45) E . (4.37)

(4.47).

(4.37) (4.40) , w -

w x t. I

326.1 0.333 0 D4.37 /F4 1 0 0 0 1 1.747 D 7470.911 D5 776 /F2 1 D F2 1 9.978 0 0 9.978 198.3257 3

$$\lambda_1(x/t) = \frac{1}{16}[4\sqrt{\rho_0} \quad 6 \quad r_3(x/t)]^2,$$

$$\lambda_2(x/t) = \frac{1}{16}[4\sqrt{\rho_0} + 2 \quad r_3(x/t)]^2,$$

$$\lambda_3(x/t) = \frac{1}{16}[4\sqrt{\rho_0} \quad 2 + r_3(x/t)]^2, \qquad (4.58)$$

w r_3 (4.48). F $1 < \rho_0$ < 4, $(x/t) \equiv 1$. ρ_0 4, E . (4.56).

H w y w . I , w . I - w , w , y . , / y y . - , w D B -E . D y .

E. Theoretical explanation of experiments

I w 2D 1D, w " y 3D , w w V_{it} [E . (3.1)] V_{ot} [E . (3.2)]. , $t=\tilde{t}$ ( ), w w ,

$$\text{2D:} \quad \Psi_{2\mathrm{D}}(\rho, t=0) = \Psi(\rho, z=0, t=\tilde{t}),$$

$$\text{1D:} \quad \Psi_{1\mathrm{D}}(x, t=0) = \Psi(x, z=0, t=\tilde{t}), \quad x \quad 0$$

$$\Psi_{1\mathrm{D}}(\quad x, t=0) = \Psi_{1\mathrm{D}}(x, t=0), \qquad (4.59)$$

C -
y y $|\Psi|^2$
y Ψ. I 3D y
[$\Psi_{3D}(r,0,t)]_r$. I 2D, y [$\Psi_{2D}(r,t)]_r$
w 1D, y [$\Psi_{1D}(x,t)]_x$. 3D
2D y w
1% w y.
w F . 23. A , F . 24
27, w 3D 1D -
y , y 1D .
I D 3D .
F , w - w
V_{it} [E . (3.1)].
$\widetilde{t}=\delta t$ (5), . N w,
y y GP w
. F 23 w
3D w $(\alpha_r=0.71)$ w $(\alpha_r=0)$ -
. D -
w w
w w ,
D
y w . O w , w y
.

F . 23 w
D y D
. y
w y .
D ($t=2.8$) .
D
y y (w
y F . 21)

E dy d BEC
d. d E -
w d - C w-
[20] w

$$\rho_t + ($$

D ... y ...
. F w , D
(w
F . 26). y
w BEC D . [38].

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