

# **Towards an Understanding of the Break-up of**







$$\sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \dots$$

So  $S'$  converges absolutely in a subset of the domain for which the original ordering converges. The domain of convergence is  $|z_1| + |z_2| < 1$ , because the double power series only “makes sense” if it converges for any ordering of its terms (and then it converges absolutely). As a final example consider the series

$$\sum_{n=0}^{\infty} \dots$$

This series converges in  $G = \{z : |z_1| < 1\} \cup \{z : z_2 = 0\}$ . The domain of convergence is the interior of  $G$ , which is  $D = \{z : |z_1| < 1\}$ . Thus  $S$  does not automatically diverge on the complement of the closure of  $D$ .

We do not have a sophisticated technique for determining  $\partial D$ . Our simple idea is to reduce the  $d$ -dimensional series to sequence of one dimensional ones. First we use absolute convergence to focus on the real series:

would have expected something like the fractal set of cusps observed for the two-harmonic standard map (Ketoja, 1989 #572); however, log-convexity would provide a strong constraint on this, so perhaps we should expect the cusps to occur only for some derivative of the boundary. There are several interesting unexplained phenomena. The domain of convergence is defined to be the intersection of the domains for



Thus there is a one dimensional unstable manifold, and a two dimensional, spiral stable manifold. The contraction on the stable manifold is rather slow.

For the general case,  $\epsilon$  is not fixed, and the amplitude map is periodically forced. However, there is still a codimension one center-stable manifold which has a one dimensional unstable manifold. On the center manifold the parameters converge to a circle on which the dynamics is a simple rotation with rotation number  $= \pi$ . The rotations arise because successive rational approximants of the incommensurate vector spiral inwards (the analogous oscillation in  $1^{1/2}$

completely defined by the configuration through  $y_t = F_2(x_{t-1}, x_t)$ . A sequence is a periodic orbit with frequency vector  $(p, q) \in \mathbb{Z}^{d+1}$  if  $x_n = x_0 + p$ , and it is a stationary point of  $W_{0,n}$  for  $0 \leq t < n$ .

Consider a generating function of the form

$$F(x, x') = \epsilon T(x, x') - V(x)$$

From the variational point of view, the map generated by  $F$  corresponds to a linear chain of particles at points  $x_j$  coupled by harmonic springs in a periodic potential  $V$ . We call the case  $\epsilon=0$  the *anti-integrable*

cantori for every incommensurate frequency near the anti-integrable limit. The converse KAM theory (Mather, 1984 #297; MacKay, 1985 #283) generalizes to a more limited extent. For  $d = 1$ , one can show that there are parame